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A Study of the Improvement in Computation of Six Children During a Six-Weeks' Term of Individual Remedial Work

Ethel Keewan Harrington
Loyola University Chicago

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A STUDY OF THE
IMPROVEMENT IN COMPUTATION
OF
SIX CHILDREN DURING A SIX-WEEKS' TERM
OF INDIVIDUAL REMEDIAL WORK

BY

ETHEL KEEVAN HARRINGTON

A Thesis Submitted in Partial Fulfilment
of the Requirements for the Degree of
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VITA

Ethel Keevan Harrington

Born in

Attended Holy Cross Parochial School, Chicago, Illinois. Graduated from St. James High School, Chicago, in June 1912. Graduated from Chicago Normal College, June 1914. Ph.B. Loyola University, 1928.

Teacher in Chicago Elementary Schools 1915-1929. Teacher of music in Chicago Junior High Schools 1929-1930. Teacher of mathematics in Chicago Senior High Schools 1930-to date.

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INTRODUCTION

During the summer of 1932 Loyola University organized special classes for remedial work in reading and arithmetic in conjunction with the Demonstration School that has been for some years a part of the summer school. Eight teachers were given charge of fifty-six pupils. Each teacher was privileged to call upon student teachers from the Demonstration School if she felt that any pupil needed more individual attention than she could give him. The children, therefore, were given close and very individual attention during the entire five-weeks' session, and the eight teachers kept complete records of all that was done, in order to be prepared to present reports on different aspects of the problems that arose. This paper attempts to present a psychological study of the improvement in computation in the fundamental processes of six pupils taught by the writer. The students worked ninety minutes daily for a period of five weeks. None of the pupils in this group was seriously handicapped mentally or physically, but the parents and teachers of each felt that remedial work in the fundamental processes was necessary.

The week preceding the opening of the term was given to testing. Each pupil was given the Binet Simon Test by an experienced tester. The children over eight years of age were examined by Dr. McMillan, physician for the Board of Education

of Chicago. The members of the arithmetic group were given the New Stanford Test in Reading: Form V and the Compass Survey and Compass Diagnostic Tests in Arithmetic. After a close study of the results of these tests the teachers felt that they were well acquainted with the pupils before the actual teaching began.

After the first week the pupils were divided into four groups on the basis of similarity of study needs evidenced by likeness in error types and identity in grade levels. As the work progressed it became evident that the retardation, in most cases, was not due to intellectual handicaps, but rather to short attention span, limited memory span, emotional disturbance, lack of individual attention, careless study habits, or a combination of some of these factors.

The teacher made an effort to teach psychologically rather than logically, to analyze the steps in the fundamental processes, and to acquaint herself with the typical error types.

At the close of the term other forms of the reading and arithmetic tests mentioned previously were administered to each pupil.

Before presenting the individual case studies, the results of recent research on the factors which should characterize remedial work are being summarized. Among those who have contributed most to this field are Leo J. Brueckner (1:337), Edward L. Thorndike (20:367) - (21:260), Frederick B. Knight

(12:63-91), and Worth J. Osborn (16:178). It has been the aim of the writer to control the desire of appearing erudite because of the citation of numerous authors, and to confine herself to a few whose contributions are of major importance.

CHAPTER I

THE LOGICAL VERSUS THE PSYCHOLOGICAL
IN ARITHMETIC

CHAPTER I

The field of arithmetic has experienced remarkable changes during the last twenty years. A readjustment and a re-evaluation of the many phases of the subject have gone on constantly. Content, teaching methods, courses of study, textbooks, and drill materials are being examined in a manner unknown two decades ago. Of all these changes, the most significant is, perhaps, that from the logical to the psychological viewpoint in teaching methods (19:21).

Until recently most of the materials of the elementary curriculum were logically organized and rigidly taught by formal methods. The mind-training resulting from the study of a certain subject was sufficient reason for the presence of that subject or topic in the curriculum. Many arithmetical processes were taught for their own sakes and not as a satisfaction of human needs. Although considerable emphasis was placed upon reasoning, an essential part of almost every exercise was memory training.

Among educators there has been a slavish demand for system for system's sake. Teachers of the more modern type maintain that this idea is only a scholar's idol. In later life the student may find it worth while to arrange his learning into a logical system. The simplicity and balance, the logical beauty

and progress and organization of a course of study are wasted on the young pupil. Very little of the symmetry and system of the older sort is left to the organization after the newer methods have made it over to suit the learner's needs. That which was one topic may be scattered over the entire course. The logical entirety of a topic may be completely destroyed by omitting a part that has been inserted merely to complete the scheme. When the faculty psychology was in vogue the logical arrangement of topics was stressed. The disciplinary rather than the practical side was emphasized. The assumption that topics, processes, and operations should be taught in a logical sequence resulted in much poor teaching. Now increased pressure is laid upon the psychological rather than upon the logical factors involved in the mastery of number facts and relations (17:16).

To permit abilities already acquired to be used in their proper connections and to allow new abilities to be applied as soon as learned a general topic that could be learned consecutively may be interrupted again and again. In the newer textbooks this type of modification occurs repeatedly. Thorndike cites the following examples. When the addition combinations with sums to 9 are well known, the pupil may be taught to use them in column addition like

3	2	3
1	3	2
<u>5</u>	<u>4</u>	<u>4</u>

and also in column addition like

23	22	21
12	31	33
<u>14</u>	<u>33</u>	<u>15</u>

before the addition combinations 5+5, 6+4, 4+6, 7+3, etc., are learned (21:88).

Multiplication combinations may be interrupted after the products of 1, 2, 3, 4, and 5 by the numbers from 1 to 10 are learned, by the introduction of the multiplication of two- and three-place number by a one-place number, for example

251	23
<u>x8</u>	<u>x2</u>

This plan permits the early application of the multiplication facts in the true connection in which they are to be used. It relieves the monotony of oral memory work at this level and of written computation later.

Part of a topic may be taken out of the place where the logical systems put it, in order that it may be put where the ability gained will notably help, or be helped by, some other ability. This is one of the two reasons for two extensive changes from the older systems:

1. Teaching subtraction along with the addition combinations.

2. Teaching each set of division combinations or "tables" along with the corresponding multiplications.

This results in:

1. Development of pupils' ability to use knowledge to gain

new knowledge.

2. Growth of power to check results in a new process.

3. Emphasis (by contrast) of the nature of each process.

To the educated adult the development of arithmetic appears to be a logical process. It seems apparent that one should begin by teaching the simple combinations first and proceeding to the more complex. There have been courses of study which required that all addition be taught first, then subtraction, then multiplication. This procedure is not in accordance with the known psychological principles by which children acquire arithmetic knowledge (34:261-262). It has resulted in the teaching of abstractions before the child has been able to give them meaning. No attempt was made to adjust the work to the maturity or to the ability level of the child.

A reaction against the logical type of teaching resulted in the development of the spiral system. The spiral mode of attack consists of treating first the simpler elements of all processes. At a higher level the elements more complex in character are considered. In the course of a few years the subject is as completely covered as in the logical method. This endeavor to build a course of study on the basis of child development has been called the psychological method. The primary emphasis has shifted from subject matter to the child and his needs.

Newer methods pay more attention to habit formation than did the older methods. The older methods assumed that pupils would reason out a certain procedure and use it without the need of special practice with it. It is not safe to make this assumption (21:64). In many cases it is true only of gifted pupils. Much of our remedial work is necessitated by too much faith in the power of transfer.

The older methods depended upon frequency of connections for habit formation (6:70). The newer methods use interest, motive, and satisfyingness. Those bonds or connections which satisfy some want or craving of the learner are formed from very few repetitions (21:57). The psychologists state two laws for the formation of mental connections. These are the laws of exercise and effect. Thorndike says that other things being equal, use strengthens and disuse weakens mental connections (21:57). Other things being equal, connections accompanied or followed by satisfying states of affairs are strengthened, whereas connections accompanied or followed by annoying states of affairs are weakened. The law of effect explains the enormous variation in the ease of learning matters which, so far as mere amount and complexity go, would be equally easy to learn. If we are to have rapid learning we must, so far as possible, get the force of satisfyingness on our side. This the newer methods try to do.

When adopting a habit to new conditions modern methods

provide against disturbance from changed conditions, making it a rule to give such help in adopting the habit to new circumstances as is feasible (21:62). Sometimes much help is needed, as in higher decade addition. The older methods ignored disturbances of this sort although they often seriously interfered with the bonds or habits concerned (21:62).

For many years arithmetic was regarded as a tool subject. The teaching of the fundamental facts was accompanied by an enormous amount of formal drill. Primary emphasis was laid upon the acquisition of skill in computation. The tables and the number combinations were often taught without relation to problems. The number facts were taught as if they were of equal difficulty.

Procedure with reference to the acquisition of skill in computation has been revolutionized within recent years. Investigations have shown that the various number combinations are not of equal difficulty, that they should not receive equal emphasis, and that their distribution throughout the course of study is a matter of importance.

No longer are the number combinations introduced into the best textbooks in an indiscriminate fashion. They are now distributed with a scientifically organized arrangement somewhat in the order of their relative difficulty. Real situations and problems provide the uses for the various combinations

One of the most vital movements looking to the greater

socialization of the materials of arithmetic is the movement for the omission of obsolete topics as well as the omission of the more purely technical ones (14:81-83). It is true and equally significant that the subject is being revitalized by the use of many out of school problems. A planned effort is being made to teach pupils those things which they will use later, and at the same time to place them in possession of that information which will enable them to understand more intelligently, and to appreciate more vividly, the world in which they are citizens.

The older system was lacking in its provision for steady review (21:68). A few isolated reviews were considered sufficient to keep the abilities alive and healthy. The newer methods set a far higher standard for a review than an occasional repeating of the same work in the same way (21:9). The newer methods seek to make reviews answer the learners' abilities and needs just as skillfully as the first learning did.

The work of pupils who are having unusual difficulties in arithmetic is carefully studied and an analysis of the specific causes is made. After this is done remedial work is prescribed to aid in overcoming the weaknesses. The study of individual cases reveals the fact that many faulty methods of work are acquired by pupils under the usual plan of mass instruction. Standards by means of which a teacher can evaluate the problems that are used in the class are in process of development.

Research has shown that the traditional notion that there

are only forty-five fundamental combinations in addition is an erroneous one. A pupil who will master all of the fundamental operations must learn more than 1300 combinations.

Not only has the educational emphasis shifted from the teaching of subject matter to the teaching of the pupil but another change is evidenced in the general effort to consider the group before the mass, the individual before the class, and actual rather than civarious experience.

Foran in the Catholic Educational Review of May, 1930, presents 60 principles from many well-known educators governing the teaching of arithmetic. The principles which show a reorganization of arithmetic instruction from the logical to the psychological are presented below:

1. The number concept is the psychological basis of arithmetic.

A large part of its development precedes the child's entrance to school. In the further development of this concept, purposeful experience with concrete objects through counting and measuring are essential. It is likely that more development occurs in the pre-school years than is usually assumed. The child's first expression of the concept is through counting.

2. The development of meaning should always precede the use of its symbols. This applies to the number-names and to the terms expressing quantities and relations such as foot, add, subtract, etc. . . .

5. Arithmetic computation includes a large number of facts and processes. The identification of these should be based upon the mental processes required rather than on the nature of the mathematical processes.

6. While some transfer of training may be expected in arithmetic, it cannot be relied upon to develop skills to the level that is required of them. Explicit training is therefore necessary in each ability that the subject involves.

7. It is unnecessary and wasteful to teach all

that is known about a process. The temptation to teach all that can be taught arises from several sources and must be controlled by definite aims. Such aims will include a definition of what is to be expected of pupils in the process in question. . . .

9. Instruction and drill should be interspersed in small amounts so that units of instruction are followed at once by drill and application. The use of large units of instruction with delayed drill creates confusion and difficulties.

10. Instruction should always be followed by drill and application. If drill precedes adequate instruction, errors are consolidated and the effect of the instruction leaves pupils in a worse predicament than they were before. . . .

14. Instruction and drill should approach processes and combinations from all angles. No important item should be presented in but a single setting. Variety contributes to interest and is necessary to insure the application of the skill under changed conditions of presentation. Pupils should receive instruction and drill in all forms of combinations such as:

4	5	4+5=	4+..= 9	9-4+..	etc.
5	4				

15. The different combinations in the four fundamental processes are not of equivalent difficulty. Instruction and drill should distribute emphasis in accordance with the difficulty of the combinations. The difficulty of each combination is partly inherent and partly personal with the pupil. Many texts do not distribute the drill in accordance with this principle and some combinations receive as much as one hundred times the emphasis accorded others. Investigations have shown that scientifically constructed drill materials yield substantially greater improvements than the same time devoted to haphazard drills.

16. There is widespread doubt about the efficacy of rules and definitions. Rules are frequently memorized without being understood. They are sometimes resorted to in lieu of adequate instruction. They tend to render procedures mechanical rather than logical.

17. Whenever rules and definitions are employed, they should be phrased in language that children understand, with only such technicalities as are necessary. Logical precision and mathematical abstractions absorb attention that should be directed to other parts of the definition or rules or to their meaning as units. . . .

21. In addition and multiplication, both forms of each combination must be taught. As far as diffi-

culty and the mental processes are concerned, $4+3$ and $3+4$ are different combinations and both require instruction and drill, as the evidence clearly indicates that a knowledge of one does not guarantee a knowledge of the other.

22. Combinations involving the same digits should be taught together. Thus, $4+3$ and $3+4$ should be taught at the same time. Each of the one hundred basic facts should lead directly to the higher-decade facts, and these should form a series which comprises a single teaching-and-learning unit.

23. Evidence indicates that addition and subtraction should be taught together rather than separately. While only one investigation has dealt with this problem, the weight of the data indicated that the together method was significantly superior to the separate method in five of the six units of the study, while in the sixth the data are open to some objections.

24. By inference but without experimental data, it appears that multiplication and division should be taught together rather than separately.

25. The rote memorization of the multiplication table as a whole is to be condemned on several grounds. Even large units from the table are unwieldy. Each number fact requires different presentations and drill. Some unconfirmed evidence indicates that giving pupils a start with the tables and then having them work out the remaining combinations through computation is advantageous. Since the combinations will appear in random order in their subsequent work, the learning of them in formal order hinders their application and easy recall.

26. All combinations must be taught in such a way that pupils will have no difficulty in using them in sequence other than those in which they were originally learned. (See Nos. 14 and 26). . . .

30. Each identified skill should be paralleled by valid drill. This principle follows from previous observations since it is necessary to have practice instruction closely with the units of instruction being short and the practice specific.

31. Drill materials should be constructed in accordance with certain specifications that have been derived from the results of experimental investigations. Most of these specifications are discussed under the headings that follow. Drill materials constructed haphazardly can scarcely fail to distribute the practice in accordance with the difficulty of the combinations.

32. Pupils vary widely in their ability to learn arithmetic, and all phases of the subject should be based on a recognition of these differences. This is particularly true of drill materials, since each pupil should practice what he does not know, not what other pupils don't know. Teaching should and can be individualized even in large classes through the use of adequate texts and suitable drill material. It is not contended that group instruction be supplanted entirely but only that pupils be afforded opportunities of meeting their own needs in so far as these are peculiar to the different members of the class. . . .

34. Practice periods should be long enough to insure adjustment to the task at hand, but not so long as to induce fatigue or monotony. The length and distribution of the practice periods are considerations that are inseparable. In general it appears that several short practice periods are superior to a few long ones, but the short ones must not be too short.

35. Drills are of at least two kinds. Some are used to create skills, while others are for the purpose of maintaining them. Drills designed to create skills should be confined to the single process with which the instruction has been concerned.

36. Drills for the purpose of maintaining skills should be of the mixed type. All four processes should be represented, and examples of the same process should be presented in different forms. This type of drill is similar in construction to the mixed-fundamental type of test in arithmetical computation. It is believed that this type of drill material is more similar to the life and problem situations involving computation than is the single process type of drill. Authorities differ on this point, but the weight of the evidence favors the mixed type of drill.

37. In the drill materials the examples should be arranged in order of difficulty. . . .

41. The drill material should afford possibilities of exact diagnosis. Diagnosis leads nowhere if it only localizes the difficulty in a major process or solely in terms of speed. The analysis of errors should be specific and permit immediate improvement.

42. Drill material should include finding errors as well as correct answers when certain conditions are fulfilled. The finding of errors is an important aspect of arithmetic ability and is used widely in checking accounts, etc. Errors should not be presented in drill material until the basic skills have been well developed. When such drill materials are employed,

attention should be drawn to the existence of errors. The finding of the errors should, of course, be followed by correcting them. . . .

45. The drill material should be in a form that will enable pupils to evaluate their progress through the year, as the motivating value of a knowledge of progress is very considerable, not only in skill but in attitudes towards the work. . . .

49. Drill work should include verbal problems which the teaching of arithmetic should not isolate from computation. It is poor teaching when computation and problem solving are kept separate. . . .

54. Problems should relate to situations that pupils can understand.

55. Problems should be real, vivid, and interesting. . . .

58. Errors belong to certain major types with which the teacher should be familiar. Classification of errors will disclose class needs and those that involve only one or two pupils.

59. Initial errors tend to become fixed unless they are overcome at the outset. Such initial errors may reassert themselves even after intervals of complete mastery of the process involved. It is therefore of special importance to prevent initial errors (34:257-266).

The recognition of these principles and their observance as a guiding influence in teaching should aid in the prevention of much remedial work now found necessary. It also seems reasonable to say that an improvement in quality and an increase in quantity of achievement of great numbers of pupils now doing work of ordinary value could be attained through the observance of these principles.

CHAPTER II

REMEDIAL PROCEDURE

CHAPTER II

The first step in remedial work is that of diagnosis. There are three types of diagnosis: general, analytical, and psychological (1:62). The general diagnosis can be made through the use of survey tests and scales. From these can be determined the general level of the pupil's ability. The general diagnosis permits the selection of pupils in need of special help.

Concerning the analytical diagnosis Leo J. Brueckner presents the following:

In this group may be placed all such diagnostic procedures involving the use of tests as are employed to determine:

(a) the particular process in which the pupil is deficient;

(b) his ability to work certain types of examples contained in sampling tests;

(c) the particular element or skill in a process which may be the cause of the deficiency;

(d) the level at which pupil mastery of a process breaks down;

(e) his ability to work a large variety of examples in a process such as are contained in comprehensive diagnostic tests (1:64).

After the analytical diagnosis the question of the cause of the difficulty presents itself. The psychological diagnosis is necessary at this point. One form of psychological diagnosis is the analysis of the written work of the pupil. Another method is to observe the pupil's mental processes in working examples. This can be accomplished by having him work aloud.

The following are steps in a psychological diagnosis:

1. Selecting of a below-standard pupil in any process or part of process.
2. Selecting a standard diagnostic test with a wide variety of types of exercises in the below-standard process.
3. Having pupil work aloud.
4. Noting verbal statements.
5. Observing habits of work.
6. Recording results.

Diagnosis reveals the fact that the necessity for remedial work may be laid to any or all of the following causes:

1. Failure of the teacher to know what was involved in the process. The teacher should be familiar with the learning process in arithmetic. In the past many of the best teachers have assumed that some facts, which are really difficult, were easily understood. Children, therefore, have been required to take big steps from a well-known process to a difficult and distantly related one (14:294).

2. Inadequate initial learning -- This is apt to occur if the teacher presents a new process without relating it to previously known and understood processes. The teacher then relies upon blind, unanalyzed drill to fix the process. In short, the comprehension of a process is secured by drilling on it instead of relating it to previous knowledge (14:295).

3. Change of emphasis in teaching -- A teacher is apt to

emphasize one process one year, and, because of a change of interest, ignore it another year. The children may suffer or benefit from this change in interest. It is right that the teacher be permitted some leeway in placing emphasis and in modifying methods. It is suggested that use of group tests and group diagnosis would be a guide to the balance of emphasis (14:295).

4. Inadequate maintenance program -- After a process is understood it should be drilled upon until a habit is formed. It is well known, that regardless of how highly mechanical skills are developed, they are soon lost unless practice in them is maintained (14:296).

5. Inadequate or poorly organized practice -- In most textbooks drills are poorly organized. Many of the exercises presented to a pupil when a new operation is in process of development are too complex. Many textbooks furnish too small an amount of supplementary material. If the teachers copy exercises from other textbooks or make up exercises the grading of these exercises according to difficulty is rarely accomplished (14:296).

Whether the story of the abilities and failures of the individual pupil is obtained by general observation or by the use of highly detailed diagnostic tests is not the vital question. The important feature of the diagnosis is that accurate and reliable data on each arithmetic skill must be obtained in

order to make possible the prescription of the proper remedy. This permits the teacher to say, "This is the nature of the pupil's difficulty." It eliminates such blanket statements as, "The child cannot divide."

Although it is generally accepted that the mental and physical growth of normal children is quite regular from year to year no evidence has yet been produced to show that the rate of growth is constant for every child (16:103). Physical growth is also subject to individual variations. "Arrested development" is the term applied to retardation in the rate of growth. It may be temporary or permanent. For this reason it is not safe to predict the future mental or physical growth of any child. While the majority of children who have a mentality rating of 80 per cent of the average for their age on the Binet Scale will continue all their lives at that level, there is a minority suffering from temporary arrest (16:103). This temporary halt may be due to poor vision, diseased adenoids, or the malfunctioning of certain glands (16:103). The discovery and removal of the trouble often permits the child to develop with sufficient speed that he makes up for lost time.

C. A. Pugsley asserts that the correction of physical defects is accompanied by noticeable improvement, not only in school achievement, but in the shortening of the time necessary to complete the eight grades (9:19). On the contrary, such authorities as Hoefer (31:) Jewett and Blanchard (22:39-56),

and De Weerdt (24:540-541) have shown that the I. Q. is relatively constant and have disproved the idea that improvement in physical condition would produce an increase in the I. Q.

Sometimes the cause of arrest in arithmetic is partly emotional (16:104). The annoying lack of success is distinctly inhibitory. It occasionally hinders thinking and learning. The child gradually comes to have a permanent feeling of inferiority with reference to arithmetic. The child's development, already obstructed by other causes, becomes more and more retarded because of the emotional element involved and finally settles down to a condition of permanent arrest. Practice produces the best results only when the results of the practice are satisfying to the individual engaged in practicing.

Another cause of arrest is boredom.(16:110). Boredom affects both men and animals, and its effects are so similar to those of fatigue as to be almost indistinguishable from them. In many schools teachers are trying to teach certain materials to pupils who are utterly incapable of learning them, while in the same grade other pupils are bored to distraction with tasks below their ability. In most of the cases of apparent fatigue the condition is that of boredom. Here is a place for intelligent motivation.

The failure to apply the principle of inference is another cause of error in arithmetic.

The ability to size up what happens under certain situations and to infer what will happen under others is of tremendous practical importance. It is the only basis upon which we can plan intelligently for the future and it is fundamental to inductive reasoning. Trouble arises because little or nothing is known concerning the ability to infer which children possess at different ages (16:114).

It is entirely normal for children who have been getting correct answers by adding to continue to add even when the type of exercise is changed. This accounts for the very large number of errors which children make through the use of the wrong process. The way to correct this type of error is to emphasize more the recognition of new elements and the presence of changed conditions.

The ability to infer and to transfer training are closely related, and these in turn are functions of native intelligence. A suggested corrective measure for errors which result from failure to infer is to reduce to a minimum the number of situations in which children in the primary grades are expected to use inference in the solution of their problems and exercises.

Lack of prerequisite experience is the cause of many errors - In many cases the teacher is responsible for this. She has not analyzed the process into steps of difficulty. The child finds himself trying to succeed in a phase of a process for which he has not been prepared.

Another factor in retardation is interference in learning.

Large numbers of children substitute something which they have previously learned for the thing which is required of them. Those who are asked to add will multiply; those who are asked to divide will subtract. This is one of the worst of all errors, and it seems to grow worse as the child progresses from grade to grade. It would not seem difficult for any one to learn that 2×3 is 6, yet continually children give 5 for an answer, because they have learned 2 plus 3 so well. In like manner there is a very marked tendency to respond "zero" to any number divided by itself. Another common type of interference is the confusion of one multiplication combination with another. In fact the wrong response is nearly always some other product of the multiplication table (16:36).

A limited attention span causes many errors in computation. As the limit of the attention span is reached, it becomes increasingly difficult to concentrate on the material at hand. The child suddenly becomes conscious of physical fatigue and of the sounds and sights around. The mind balks at the next addition. It finally becomes imperative that the child momentarily interrupt the adding activity and attend to something else. If this is done for the fraction of a second, the mind may clear and the adding activity may continue smoothly for a second group of figures. Very little is known about bridging attention spans. One suggestion is to repeat mentally to oneself the sum during the interval of interruption. This pause is used to rest the mind through the monotony of the repetition of the sum (11:26).

A limited memory span is a factor in retardation. Teachers must consider the four phases of memory in their arithmetic teaching. These phases of remembering are formation, retention,

recall, and recognition. Connections made for the first time are likely to be more permanent (16:117). Connections accompanied by vivid or striking feelings are likely to be lasting (16:117). Single connections are more permanent than those of the multiple type.

To attempt to present more than one thing at a time is trying to do two things at once. Probably neither will be done well; for example, it is bad policy to teach two methods of subtraction at once.

Recall is strengthened when connections are closely associated with situations similar to those the child will meet most frequently in daily life; during vacations, and in after life. Hence childhood will profit tremendously from a small amount of intelligent review that which they knew the previous year (7:111).

Many textbooks ignore the fact that combinations in the various processes are not of equal difficulty (34:260). Some combinations require little drill before they become fixed while others require extended repetition before they become automatic. The amount of drill required also varies with the child. Investigations indicate that the scientifically constructed drill materials yield substantially greater improvement than the same time devoted to haphazard drill (2:116). If the drill material is not adequate and sufficiently motivated the teacher will find it almost impossible to keep close

account of the learning of the pupils. Disastrous underlearning of many bonds is almost sure to occur and hinder the progress. One of the most far-reaching causes of incorrect habits of work in arithmetic is the application of drill exercises on processes which the pupil has not yet learned to do (34:258). Under this condition drill exercises tend to "set" the bad habits of work and do more harm than good. Only correct drill brings good results; incorrect drill makes matters worse. Regardless of the value of drill materials or their scientific distribution, their advantages will not reach the pupil if methods of work are incorrect.

In the Second Year Book of the Department of Elementary Principals Elda Merton has presented the steps of each of the fundamental processes. It seems fitting to insert them at this point. Much remedial work could be eliminated entirely if the teachers realized the number of steps of each process involved and the sequence of their steps.

The specific abilities listed for addition are as follows:

1. The 100 addition combinations.
2. Ability to apply the combinations to higher decades:

$$\begin{array}{r} 2 \\ 4 \\ \hline 52 \\ 4 \end{array}$$
3. The meaning of the addition sign.
4. The meaning of the following terms: Addition, add, addend, and carrying.
5. That in writing the example, units must be placed under units, tens under tens, etc.
6. That one must begin at the right and work to the left.
7. That unit figures should be added to unit figures in a column, tens to tens, etc. This also includes the ability to keep one's place in a column.

8. Ability to add a "seen" to a "thought of" number:

47

56

93

After a child has added 3 and 6, he no longer sees both of the numbers he is required to add. Now 7 is a "seen" number, but 9 is only a "thought of" number. This also includes the ability to keep in mind the result of each addition until the next number is added to it.

9. How to regard zeros in a column.
10. How to regard empty spaces in a column.
11. How to place the answer in the sum when a column has been added and the total sum of these figures is less than 10.
12. How to proceed with the next column when meeting the condition in 11.
13. How to place the answer in the sum when a column has been added and the total sum of these figures is 10 or more than 10.
14. How to proceed with the next column when meeting the condition in 13; i.e., carrying.
15. Ability to remember the number carried.
16. How to proceed when the need for carrying and no carrying is met alternately in an example.
17. How to place all the numbers in the sum.
18. How to check for correct answers.
- 19.

An analysis of the skills and knowledge needed in subtraction is as follows:

1. The 100 subtraction combinations.
2. The ideas in one's subtraction concept:
 - Taking away idea: 15-7; 7 from 15
 - Adding idea: What number added to 7 equals 15?
 - Difference idea: 15 is how many more than 7?
3. The meaning of the following terms: Minus, less, subtract, minuend, borrowing, difference, remainder.
4. The meaning of the subtraction sign.
5. That the complete minuend must always be larger than the complete subtrahend.
6. That in writing the example, units must be placed under units, tens under tens, etc.
7. That in writing the example, one must begin at the right and work to the left.
8. That the order of units in the subtrahend must be subtracted from the same order in the minuend.
9. How to proceed when the first number to be subtracted in the minuend is larger than the corresponding number in the subtrahend.
10. That one must now borrow unless the number in the subtrahend is larger than the corresponding number in the minuend.

11. How to proceed when a number of the subtrahend is larger than the corresponding number of minuend; i.e. borrowing.
12. What it means to place a 1 in front of a number when borrowing ten:

$$\begin{array}{r} 423 \\ -219 \\ \hline \end{array}$$

13. What it does to the next number in the minuend when a 1 has been placed before the following number.
14. Must be able to remember the new number made through borrowing:

$$\begin{array}{r} 628 \\ -239 \\ \hline \end{array}$$

After subtracting 9 from 18 the child is dealing with 11, not 12.

15. How to proceed when the need for borrowing and no borrowing is net alternately in an example.
16. How to borrow when two or more successive digits in the subtrahend are longer than the corresponding digits in the minuend.
17. How to proceed when there are fewer figures in the subtrahend than in the minuend.
18. How to proceed when the last subtraction takes places with the subtrahend and minuend the same:

$$\begin{array}{r} 649 \\ 623 \\ \hline \end{array}$$

The zero must not be placed in the remainder.

19. Ability to handle a zero or a succession of zeros in the subtrahend.
20. Ability to handle a zero or a succession of zeros in the minuend.
21. How to check for correct answers. (10:396).

The basic knowledge needed in multiplication is as follows:

1. The multiplication tables through 9×9 including zeros.
2. How to add.
3. The meaning of the multiplication sign.
4. The meaning of the following terms: Multiplication, product, multiplicand, multiplier, carrying, and sum.
5. That in writing the example, units must be placed under units, tens under tens, etc.
6. That the multiplier is always a number of
7. That the number in the multiplicand is to be multiplied by the numbers in the multiplier.
8. That one must begin at the right and work to the left.
9. How to place the product after the first multiplication when the product is less than 10.
10. Ability to proceed with the multiplication of the next digit when meeting the condition in 9.

11. How to place the answer in the product when the product is 10 or more than 10.
12. How to proceed with the multiplication of the next digit when meeting the condition in 11; i.e., carrying.
13. Ability to remember the number carried.
14. Ability to _____ and _____ quickly.
15. Ability to handle zero or a succession of zeros in the multiplicand.
16. How to proceed when the need for carrying and no carrying are met alternately in the example.
17. How to proceed after the multiplication by the units figure of the multiplier is completed when there is more than one figure in the multiplier.
 - (a) Which number to multiply by next.
 - (b) Where to place the first product.
18. Ability to handle a zero or a succession of zeros in the multiplier.
19. That these products must be added and how this is done. This involves any or all of the 18 points on addition.
20. How to check for correct answers (10:397).

The Knowledge needed for short division follows:

1. The multiplication tables through 9×9 including zeros.
2. The 100 subtraction combinations.
3. The division tables through the 9's including zeros.
4. The meaning of the signs for division, $\frac{\quad}{\quad}$, $\overline{) \quad}$.
5. The meaning and use of each of the following terms: Division, product, dividend, carrying, divisor, quotient, remainder.
6. How to proceed when the first number of the dividend is the same as, or greater than, the divisor.
7. Where to place the first number of the quotient when meeting the condition in 6.
8. Must know how to proceed when the first number of the dividend is smaller than the divisor, as $4 \overline{) 104}$.
9. Where to place the first number in the quotient when meeting the condition in 8.
10. Each step:
 - (a) Divide
 - (b) Place quotient figures
 - (c) Multiply
 - (d) Subtract
 - (e) Carry
11. How to handle the remainder after subtracting: $2 \overline{) 9154}$.
12. Ability to remember the large number carried.
13. That no number equal to or larger than the divisor can be carried.
14. Ability to find the correct quotient figure with a minimum of trial; i.e., rapid recognition of the two factors, one being given.

15. How to continue dividing after some number in the quotient brings no remainder: 18
2 364
16. Ability to handle the zero in the quotient and make the proper operation in the dividend: 15
4 6012
17. Ability to handle the zero in the dividend when alone or used with a number carried.
18. How to handle a zero or a number of zeros at the end of the dividend.
19. How to place correctly all quotient figures.
20. How to handle the remainder at the end of a problem that does not "come out even."
21. How to check for correct answers.

The additional basic knowledge needed in long division follows:

1. Ability to subtract under conditions found in the long division process.
2. Ability to estimate quotients of all types, chiefly in examples with two- and three-place divisors.
3. How to multiply and carry.
4. The steps in the process:
 - (1) Divide
 - (2) Place quotient figure correctly.
 - (3) Multiply divisor and correct quotient figure
 - (4) Place the resulting product correctly.
 - (5) Compare the number from which it is being subtracted.
 - (6) Subtract.
 - (7) Compare remainder and divisor.
 - (8) Bring down digit from dividend.
 - (9) Continue these steps until the example is completed (10:398).

Each of the fundamental operations has its own pitfalls. Some error types are common to all processes. The zero difficulty exemplifies this statement. Dr. Orborn from an extensive survey of city, town, and village children of many nationalities and levels of intelligence concluded that errors are typical and not haphazard in character (16:32). G. T. Buswell and Lenore John have made notable contributions to this phase of remedial procedure. Their diagnostic charts contain the

error types for each process. For this study the writer is listing some of the errors suggested by Leo J. Brueckner, and also found in the case studies of this report.

Those found in addition follow:

1. Weakness in combinations.
2. Counting
3. Vocalization
4. Adds same digit to both columns
5. Bridging
6. Zero difficulty
7. Breaks up combinations
8. Roundabout methods of work
9. Carrying difficulties:
 - (1. Forgets to carry.
 - (2. Adds carried number irregularly.
 - (3. Carries wrong number.

Those found in subtraction follow:

1. Weakness in combinations.
2. Counting.
3. Zero difficulty.
4. Borrowing difficulty:
 - (a) Not allowing for borrowing.
 - (b) Failure to borrow, giving zero for answer.
 - (c) Deducting from minuend when borrowing is not necessary.
5. Subtracting minuend from subtrahend.
6. Roundabout methods of work.
7. Skipping one or more decades.

Those found in multiplication follow:

1. Weakness in combinations.
2. Counting.
3. Zero difficulties.
4. Carrying difficulties.
5. Errors in adding.
6. Errors in multiplying.
7. Omits digit in multiplier, multiplicand, or product.
8. Uses wrong process.
9. Errors in position of partial products.

Those found in division follow:

1. Weakness in combinations.
2. Difficulty with remainders.
3. Zero difficulty.
4. Difficulty with quotient.
5. Roundabout methods of work.
6. Difficulty with subtraction.
7. Difficulty with multiplication.
8. Repeats tables for results (1:72-87).

Present emphasis on remedial work is a reflection on the failure of teaching arithmetic. Previous faulty procedures have produced difficulties. Probably no teaching scheme will eliminate remedial work, but it should be reduced to a minimum. Prevention is better in every way than remedy.

It is probably that success in prevention work lies not in

increasing the quantity but in improving the quality of drill work from the beginning. If errors are due to the wrong experiences and lack of experience in systematic drill then the main drive upon remedial work should be in the reconstruction of drill content. There is some ground for the contention that much of the remedial work will no longer be needed when drill work meets more adequately the following specifications:

1. Drill should be on the entire process.
2. Drill should come frequently and in small amounts.
3. A drill unit should be a mixed drill.
4. Drills should possess time limits.
5. Drills should use accuracy standards.
6. Examples in a drill should come in order of difficulty.
7. Drill work should include verbal problems.
8. Drills should facilitate the diagnosis of specific error for the facilitation of remedial work (12:64-65).

Development of effective remedial drill in arithmetic depends upon the accuracy with which the various skills of the different processes are isolated and identified. Remedial work functions only when the exact level at which pupil mastery breaks down has been located.

Correct skills should be identified and paralleled and all phases of the skill covered. Remedial drill in any process should cover all situations in that process; and also the most important variants of each situation.

Remedial drill must cover in a valid manner all the numerous underlying skills and it must also provide the means for bringing about the gradual knitting together of the components into the complete function.

Valid drill material must be provided for each specific drill. The validity of the drill depends to a certain degree upon the validity of the analysis of skills. Good luck or haphazard methods are effective only by chance.

One hundred per cent validity of drill material may be achieved only by taking a sampling of one hundred per cent of all the possible facts. This is rarely possible, but a large enough sampling to include the most frequently used facts may be obtained. Alternate drill materials which appear on surface to be practically identical may be quite different in actual potency. Construction of drill material of value requires a measure of expert knowledge, patience, and accuracy.

The last word in remedial work in arithmetic will not be spoken until many studies on that subject have been based upon students who have had proper drill work from the beginning. It is not known at present what errors are due to true learning difficulty and what errors are due to faulty drill construction. If the teacher notices the type of example missed most frequently she can make a good guess as to what total process needs special drill the most.

We should not be concerned with the planning of remedial

work for pupils who have not had the correct experience with numbers in their systematic instruction and drill. We should be concerned with remedial work for errors which persist after the pupil has had the best type of drill available. To summarize, therefore, what has been said in this chapter, we may conclude by stating that good remedial work will be characterized by the following qualities:

1. The discovery of weakness.
2. The discovery of exact level of this weakness in the particular skill under study.
3. The location of the cause of the difficulty.
4. The formulation of remedial exercises which attack this cause.
5. The enlistment of the pupil's cooperation.
6. The measurement of progress.
7. The adjustment of work to changing needs until the removal of the difficulty.

CHAPTER III

CASE STUDIES

Case Study of G. B.

George had finished fourth grade when he entered summer school. His age was ten years and three months. Records of his physical examination indicated that George was a mouth-breather, and nose-bleeder. His hearing and vision were considered normal. His appetite was variable and fussy. He was constipated. The doctor recommended that a special examination be given to his nasal condition.

A preliminary test in arithmetic revealed weakness in subtraction, multiplication, and division. The first two deficiencies mentioned were partially responsible for the third. The total score of this test gave George an educational age of nine years and a grade equivalent of low-fifth. George had an I. Q. of 94.

The pupil's parents were foreign. Broken English was spoken at home. George was docile and obedient in an unthinking, dependent manner which the writer has found characteristic of many foreign.

As the summer term progressed it was discovered that George had an aversion to checking; that problem-solving ability was retarded because of language difficulties; that any pressure, even though very slight, upon the development of speed, produced disappointing results in the quality of the work.

Although the physical examination indicated that he posses-

sed normal vision there was a confusion of numbers which were close together. Misspellings of words which were copied from a book may have been due to a very slight deficiency, but a great many were the results of careless reading. Misspellings of the following copied words occurred through the term: arithmetic (arithnetic), multiplication (multicpation), oranges (oronges), greater (gretter), marbles (marbeles), dollar (doller), and March (Marck).

Much of the written work in computation had a comparatively high rate of occuracy. The oral responses were often incorrect and always slow. The language difficulty had some effect on this condition. Lack of automatism in the combinations of the four processes adversely affected the readiness and accuracy of these responses.

During the term George had daily opportunities for study, oral recitation, restudy, written work in problems, in combinations of four processes, and in examples involving the four fundamental processes. A work book, Intermediate Pilot Arithmetic, and Elementary Problem Book were the texts used.

A comparison of the results of the preliminary and final tests does not show the growth made by George in concentration, industry, habits of organization, and in checking ability. There was a slight progress in paragraph comprehension for which the problem study may have been partly responsible.

This general description will be followed by a daily study of George's errors, accuracy of achievement, and growth in helpful habits.

Table I gives data which show the student's daily accomplishment, his accuracy, and his percentage of accuracy during the period of remedial work. A consistent effort was made to keep the work in the fundamental operations well balanced. The problem work was also carried on with regularity.

As mentioned before, the student's percentage of accuracy was, in general, reasonably high. His work revealed no outstanding errors in any process. George was the type of student whose work was effected radically by outside influences. Fatigue and extreme heat influenced his score adversely several days during the term. He was apparently phlegmatic rather than emotional in type. Some inner conflict may have caused the occasional failures in a process which had merited 100 per cent accuracy for several preceding days. There seemed to be an instability of I. Q. which will be discussed in another portion of this paper.

In the Compass Survey Test, Advanced Form A, the following results were obtained:

In the addition of the preliminary test all attempted were correct. In subtraction three of six attempts were correct. The three worked incorrectly were not wrong because of errors in the subtraction combinations nor in the subtraction process.

TABLE I

Number of Problems Attempted by George, Number Right,
and Percentage of Accuracy

	Addition			Subtraction			Multiplication			Division			Problems		
Preliminary	6	5	83.3	5	3	60	6	3	50	4	1	25	0	0	0
27															
28	12	11	91.6	27	27	100	12	12	100						
29	14	12	85.7										4	3	75
30	4	4	100	4	4	100	6	4	66.6				2	2	100
1							9	3	33.3						
5				12	12	100				6	6	100	6	2	33.3
6				12	12	100	4	4	100	4	4	100	5	5	100
7				10	10	100	5	5	100	8	8	100	5	4	80
8				6	6	100	5	5	100	8	8	100	5	4	80
11	110	105	95.5							36	19	52.7			
12	64	55	85.9	56	56	100				27	12	44.9			
13				46	46	100				12	7	58.3	5	3	60
14	6	6	100	18	17	94.4							6	5	83.3
15				18	18	100				38	36	94.7	7	7	100
18							146	142	97.2	8	6	75	11	10	90.9
19										16	10	62.5	8	7	87.5
20							5	5	100	10	10	100	10	10	100
21							5	5	100	10	9	90	10	4	40
22	58	52	89.6	51	50	98	51	46	90.1	14	14	100			
25	2	2	100	2	2	100	51	49	96	2	1	50	5	5	100
26	2	2	100	2	2	100	2	2	100	10	10	100	16	11	68.7
27	56	55	98.2				20	19	95	5	5	100			
28	28	27	96.7	28	28	100	28	25	88.8	28	20	71.4			
Final	5	4	80	5	4	80	4	3	75	3	1	33.3	2	1	50

TABLE II

Opportunities of George for Errors of 17 Tapes; by Days;

Number of Errors Made; and Percentage of Accuracy

Error	1	2	3	4	5	6	7	8
Preliminary	1 1 100	2 2 200	1 - -	4 4 100	1 - -	2 2 200	7 1 14.2	12 8 3.5
27								
28			1 36 2.7					
29								
30								
1								2 45 4.4
5								
6								
7								
8								
11							2 36 5.5	
12			41 53 2.6				1 51 1.9	
13								
14								
15								
18							1 8 12.5	
19								
20								
21							1 14 7.1	
22			1 35 2.8					3 112 2.6
25								5 98 5.1
26								
27								1 83 1.2
28								3 98 3.0
Final								

TABLE II (continued)

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	9	10	11	12	13	14	15	16	17
Preliminary									
27									
28									
29	2 11 18								
30	1 40 2.5	1 3 33.3	1 1 100	1 1 100					
1		1 4 25	1 16 6.2	1 1 100					
5									
6									
7									
8									
11					14 36 38.6	8 28 28	5 110 45		
12	4 43 9.2						1 22 45		
13					4 32 12.5				
14									
15					1 34 2.9		2 25 8		
18					2 20 10				
19	2 4 50				5 24 20.8				
20									
21					1 33 3.0				
22	3 54 5.5						4 107 3.7	2 36 5.5	
25	5 98 5.1								
26									
27							2 85 2.3		
28	1 19 5.2				8 64 12.5		1 44 2.2		
Final					2 7 28.2		2 20 10		

Two were due to omission of a decimal point; one was due to ignorance of the method of subtracting denominate numbers.

In multiplication three of six attempts were correct. One error was due to ignorance of multiplication process with fractions; another occurred because of a mistake in multiplication combination; the third was caused by an apparent oversight. The multiplying of the last number in the multiplicand was omitted.

In division one of four attempts was successful. The first error in a short division example was a subtraction error; correct quotient but incorrect remainder. The other two attempts were made in long division examples. The student seemed to have no knowledge of long division form. There was an attempt to estimate the number of times the "tens" digit was contained in the quotient. These estimations were incorrect.

No attempt was made to work percentage or the general problems. George's grade and age would indicate that three of the general problems could have been solved. The percentage was naturally beyond him.

As eight of the seventeen errors found during the entire term appeared in the preliminary tests all error types and a sample of each will be listed below.

1. Failure to place decimal point.

$$\begin{array}{r} \$15.00 \\ -3.89 \\ \hline 11.11 \end{array}$$

2. Ignorance of the process of subtracting denominate numbers.

$$\begin{array}{r} 2 \text{ yd } 1 \text{ ft } 6 \text{ in} \\ -1 \text{ yd } 2 \text{ ft } 8 \text{ in} \\ \hline 8 \text{ ft } 8 \text{ in} \end{array}$$

3. Error in adding a "seen" to a "thought of" number.

$$\begin{array}{r} 22 \\ 963 \\ 45 \\ 886 \\ 378 \\ \hline 2252 \end{array}$$

4. Ignorance of process of multiplying fractions. $\frac{2}{3} \times 10$

5. Unintentional omissions.

6. Ignorance of long division form $\frac{9.99-9}{21 \overline{17.85}}$

7. Wrong remainder after selecting correct quotient

$$\begin{array}{r} 064-3R \\ 7 \overline{450} \end{array}$$

8. Errors in multiplication combinations 7×2

9. Failure to add carried numbers.

$$\begin{array}{r} 42 \\ \times 28 \\ \hline 326 \\ 84 \\ \hline 1166 \end{array}$$

10. Carried wrong number.

$$\begin{array}{r} 975 \\ \times 34 \\ \hline 3900 \\ 2825 \\ \hline 32150 \end{array}$$

11. Error in higher decade addition. $18+8 = 28$.

12. Ignorance of process of multiplying by hundreds.

$$\begin{array}{r} 19 \\ \times 400 \\ \hline \end{array}$$

13. Error in selecting correct quotient number. $\frac{171-2}{4 \overline{756}}$

14. Error in writing numbers.

For 6870 George wrote 6817.
 " 2060 " " 2016.

15. Error in addition combinations. $9\div 8$ $7\div 9$ $9\div 5$.

16. Forgot that he had borrowed: $\$7.00$
 $\underline{3.07}$
 $\$4.93$

17. Error in higher decade combinations: $14\div 6$ $35\div 6$.

Following is the record of daily accomplishments referred to on page two of this paper:

June 27

This day's assignment involved the combinations of the second, third, and fourth quadrants.

Twenty-seven multiplication examples had one digit multipliers. The paper had no errors.

June 28

Eleven were correct of twelve addition examples with three four-digit columns each.

Tables of 6 were written backwards correctly.

Twenty-seven were correct of twenty-seven two-column subtractions involving borrowing.

June 29

Twelve were correct of fourteen multiplication examples with multiplicand and multiplier having 2 digits each.

Three were correct of four problems. The error was one of reasoning.

June 30

Four of four simple subtractions were correctly worked.

Four of four simple addition examples had 100 per cent accuracy.

Two or two problems were solved and computed correctly.

Four of six multiplication examples were correctly computed; five of these had two-digit multipliers. One had a multiplier of 200. The student wrote a note telling the teacher that he could not do this kind.

July 1

Only three of nine multiplication examples were worked correctly. Most of the errors occurred in carrying.

July 5

Twelve of twelve two column subtraction examples received a grade of 100 per cent. Six of these examples involved borrowing.

Six of six short division examples were computed with 100 per cent accuracy. The divisors were 4, 5, 6, 7, 8, and 9.

Two of six problems were solved correctly. Three mistakes were made in the selection of the process. The remaining mistake was an oversight. George neglected to multiply the last number of the multiplicand by the multiplier. Many erasures throughout the paper indicated that upon checking the first responses were changed.

July 6

There were twelve correct of twelve simple two-column subtraction examples.

Five of five simple one-step problems involving simple subtraction and addition were computed accurately.

Four of four multiplication examples involving combinations of the second and fourth quadrants received a mark of 100

Four of four short divisions were correct.

July 7

Ten of ten three column subtractions involved carrying in two consecutive column were computed correctly.

Four of five problems were correctly solved. The error was made in the selection of the process.

Eight of eight short divisions with divisors ranging from 2 to 9 received a grade of 100 per cent.

There were five correct of five one-digit multiplier examples with multipliers of 6, 7, 8, 9, and 2 respectively.

Six of six one-step problems in subtraction were graded 100.

July 8

Four were correct of five problems in which fundamental processes were varied. The incorrect one was incomplete.

In eight short division examples all were correct. The divisors were 2, 3, 4, 5, 6, 7, 8, and 9. The answers involved zeros in the quotient.

In five multiplication examples involving combinations of the second and fourth quadrant the mark was 100.

July 11

Nineteen of thirty-six simple short division were correct. Thirty had remainders.

The divisor was 4 in all cases. The student thought there was a relationship between the answers. He simply wrote them on an ascending scale as follows:

$$\begin{array}{cccccccc}
 4 \overline{) 36} & 4 \overline{) 37}^{8-1} & 4 \overline{) 38}^{8-2} & 4 \overline{) 39}^{8-3} & 4 \overline{) 17}^3 & 4 \overline{) 18}^{3-1} & 4 \overline{) 19}^{3-2} & 4 \overline{) 24}^{3-3} \\
 4 \overline{) 25}^4 & 4 \overline{) 26}^{4-1} & 4 \overline{) 27}^{4-2} & 4 \overline{) 33}^{4-3r} & 4 \overline{) 34}^5 & 4 \overline{) 35}^{5-1r} & &
 \end{array}$$

Another difficulty arose in the following combinations:

$$8 \times 4 = 36 \quad 3 \times 4 = 12 \quad 4 \times 4 = 25$$

Three sheets of a workbook were completed with the following results:

Attempts	Correct
55	54
55	51
28	20

Papers one and two included simple addition combinations. The five errors included the following combinations:

$$\begin{array}{ccccc}
 7 & 7 & 9 & 9 & 9 \\
 2 & 9 & 5 & 6 & 8 \\
 \hline
 8 & 15 & 15 & 12 & 14
 \end{array}$$

The third paper covered material in writing numbers. Errors were not due to ignorance of place value. Errors

occurred in the last two digits as indicated:

Two thousand sixty was written 2016.

Six thousand eight hundred seventy was written 6817.

Seventy-five thousand eight hundred fifty dollars was written \$75815.

If these numbers had been dictated one would be inclined to assume that the similarity in sound of sixteen and sixty, seventeen and seventy occasioned the errors. As the work was entirely written it may have been George's usual inclination to handle the last portion of any operation carelessly.

July 12

George worked twenty-seven examples in division. Twelve were wrong. Eleven were incorrect because George discovered the answers on the opposite page. His discovery didn't carry him far enough, as he copied the answers from the wrong place. Several of those correctly worked were as difficult as those. A little talk with George convinced him that he was fooling only himself. One error was due to lack of subtracting ability.

In fifty-six simple subtraction combination the mark was 100. Eighteen of eighteen examples adding without carrying were correct. Fifteen of eighteen examples involving adding with carrying were correct.

Twenty-two of twenty-eight addition examples were computed correctly. These examples included:

eight three-digit one-column additions;
 eight four-digit two-column additions;
 six five-digit two-column additions;
 six five-digit three-column additions.

Two of these examples had irregular columns.

July 13

Three of five problems were correct. The mistakes were of reasoning rather than of computation.

The twelve examples for which George had copied answers on the previous day were taken from the workbook and worked on a regular paper. Seven were correctly finished. The five incorrectly worked had divisors of 5, 7, 8, 9, and 9 respectively. The teacher gave oral drill on these combinations and showed him how to put down the products and subtract from the part quotient in the following fashion:

$$\begin{array}{r} 6 \\ 9 \overline{) 331} \\ \underline{036} - 7r \end{array}$$

$$\begin{array}{r} 33 \\ \underline{27} \\ 6 \end{array}$$

$$\begin{array}{r} 61 \\ \underline{-54} \\ 7 \end{array}$$

George selected the wrong quotient number 4 times. He omitted one intentionally.

The division combinations related to the multiplication combinations of the fourth quadrant were troubling George. Forty-six of forty-six subtraction combinations were worked correctly.

July 14

Five of six simple problems were correctly worked. The

error was committed in the selection of the incorrect process.

Seventeen of eighteen subtraction examples without borrowing were correct. The teacher believes the error was due to a visual weakness mentioned before. This was the example:

$$\begin{array}{r} 86 \\ 52 \\ \hline 37 \end{array}$$

George evidently combined the five and two of the subtrahend. Six addition examples with long two-and three-digit columns were correctly worked.

July 15

Seven one-step problems involving simple manipulations of three of the fundamentals were correct.

Eight of ten short division examples were correct. The wrong quotient number was selected once. An example with 8 for a divisor was intentionally omitted.

In a subtraction test, a mark of 100 was earned. There were eighteen examples.

In a test of subtraction of money numbers there were three intentional omissions. Two mistakes concerned a new error. George forgot that he had borrowed.

An added difficulty in these problems lay in the fact that the minuend in each case ended with two zeros. If checking had occurred George would have cleared these mistakes.

Twenty-eight division combinations, using 7 as a divisor, were answered correctly.

July 18

On eighteen multiplication examples involving no carrying and with multipliers ranging from 2 to 9 George made 100 per cent. Of eighteen multiplication examples involving carrying, and with multipliers ranging from 2 to 9, sixteen were correct. One error occurred in the multiplication combinations. The other occurred in carrying the wrong number.

One problem, involving multiplication, was correctly worked.

One hundred eight of one hundred ten multiplication combinations were correctly worked.

Nine of ten problems were correctly worked. The incorrect one was intentionally omitted.

Six of eight short division examples with divisors ranging from two to eight were worked correctly.

There were two errors in one example. George chose the wrong remainder after first quotient number was selected. The second quotient number was incorrect. With the second example the wrong quotient number was selected.

July 19

George was given a checking test in division. He was to decide whether, in four examples, the first number in the quotient was correct. He checked four correctly. He was then given four examples to check in relation to the second number in the quotient. Two of these were marked correctly.

George had been told about the proper form in long division. He was given eight easy division examples, resulting in two digit quotients with no remainders. Four of these were correctly worked.

Two errors occurred in failing to add a carried number. One example was intentionally omitted. A third error revealed an ignorance of how many 12's are in 114.

Seven of eight problems were correctly worked. The incorrect one revealed a lack of reading ability. The computation was correct.

July 20

Ten of ten problems were accurately worked.

Five of five multiplications involving third and fourth quadrants were graded 100 per cent.

Ten of ten short divisions (divisors 2-9) were perfectly finished.

July 21

In ten problems three were omitted intentionally. Four were correct. Three were correct in computation, but incorrect in process selection.

Five of five multiplications (with one digit multipliers) were correct.

Nine of ten short division examples were correct.

July 22

In a mixed fundamental drill of six more difficult

examples, including addition, subtraction and multiplication, George had three right and three wrong.

In the two addition examples the error occurred in adding a "seen" to a "thought of" number.

In two multiplication examples (with multipliers of three digits), George forgot to carry two times. He evidently left the carried number till the last, for it was often forgotten even though it had been written.

In a multiplication example (with a two-digit multiplier), there was an error in combinations.

Fifty-one of fifty-six additions of higher decades were perfectly finished. Four errors occurred in addition combinations. One occurred in failure to remember the carried number.

George worked forty-nine one-digit multiplier examples involving third-and fourth-quadrant combinations. Forty-five were worked correctly. Two errors occurred in addition of higher decades. Two occurred in multiplication combinations.

In a subtraction combination drill forty-nine of forty-nine combinations were correct.

Fourteen of fourteen easy long division examples were correct. The divisors were 40, 20, 50, 32, 60, 42, and 63.

July 25

Five of five one-step problems were accurately solved. A mixed drill in fundamentals brought the following results:

Two examples of:

Addition of three six-digit column merited 100 per cent

Subtraction of $\begin{matrix} \text{(four column minuend)} \\ \text{(three column subtrahend)} \end{matrix}$ merited 100 per cent.

Multiplication $\begin{matrix} \text{(Multiplicand three digit)} \\ \text{(Multiplier two digit)} \end{matrix}$ merited 100 per cent.

Long Division (Divisors 33, 93, 52, and 59) merited 100 per cent.

The errors in long division occurred in the adding of a carried number when multiplying quotient digit by divisor.

In a practice in multiplication combinations 47 of 49 were correct.

July 26

Ten short divisions with divisors ranging from 2 to 9 were worked correctly. Obtaining the correct answers involved the placing of zeros in the quotient and much carrying and subtracting.

Seven of ten problems were worked correctly. One was omitted entirely and two were correctly computed, but the wrong process was selected in each case.

In a mixed fundamental drill 100 was obtained on each process.

Four of six problems were worked correctly. Two were incorrect because of desire to manipulate figures rather than to use judgment.

July 27

On this day George was given twenty examples involving the multiplication of money. Only one error occurred in the placing of the decimal point. However, there was one error in multiplication combinations, one error in the addition combinations, and two errors because of failure to add the carried number.

In fifty-six addition combinations George had one error.

In five long division examples George made 100. The multiplication examples involved in the proofs were also correct.

July 28

In a mixed fundamental drill of six examples George made 100. An error occurred in but one example. It was a recurrence of an error made some time before.

In fourteen examples from the Buswell and John Test in Fundamentals one error in an addition combination was noted.

In the same test twenty-eight of twenty-eight graded subtraction examples were graded 100.

In multiplying, twenty-five of twenty-eight examples were correct. Two of the errors occurred in combinations. One error was the failure to add the carried number.

In long and short division twenty of twenty-eight examples were correct. The eight errors were made in selection of the quotient number.

The final test used was the Compass Diagnostic, Form B: Advanced. The following results were obtained:

	Total Problems in Test	Attempts	Correct
Addition	10	5	4
Subtraction	10	5	4
Multiplication	10	4	3
Division	10	3	1
Percentage	10	0	-
General Problems	10	2	1

The teacher felt that at the end of the summer term George still showed an urgent need for intensive work in English, spelling, and reading. George's improvement was not so tangible as that of some of the other pupils. However, the summer work produced a better attitude toward industry in a desire to work to capacity.

The record of his standing in the initial and final tests in arithmetic and reading appears below. Blank spaces indicate that the student's educational age and grade equivalent could not be secured from his scores.

Arithmetic Test Scores, Educational Age, and Grade
Equivalent of George in Initial and Final Tests

Processes	Compass Survey Educational Age Grade Equivalent					
	Initial	Final	Initial	Final	Initial	Final
Addition	6	4	10-6	9-0	H-5	H-4
Subtraction	3	4	9-0	9-6	-	H-4
Multiplication	3	3	9-6	9-6	L-5	L-5
Division	1	1	-	-	H-3	-
Percentage	0	0	9-0	9-0	-	-
Problems	0	1	-	-	-	-
Total	13	13	9-0	9-0	5-0	5-0

Reading Test Scores, Reading Age, and School
Grade of George in Initial and Final Tests

	New Stanford Reading Age School Grade					
	Test		Reading Age		School Grade	
	Initial	Final	Initial	Final	Initial	Final
Paragraph Meaning	46	49	9-8	9-11	3.9	4.0
Word Meaning	47	47	9-9	9-9	3.9	3.9
Total (average) Reading	46.5	48	9-8	9-10	3.95	4

Observation of George at work and a detailed study of his daily papers suggested to the writer the following conclusions:

1. A time limit adversely affected the accuracy and quantity

of his achievement.

2. Incorrect answers were as satisfactory to George as the correct answers were. This may account for the consistent omission of the carried number, in multiplication and addition.

3. Satisfaction with a response, correct or incorrect, may have been a reason for the student's aversion to checking.

4. Lack of power in work-type reading influenced the problem work adversely.

5. As many of the errors occurred at the end of the example a limited attention span may have been their cause.

6. A limited memory span may have caused the lack of automatism in the multiplication combinations.

George is a most interesting study in mediocrity. Mentally and educationally he was perfectly normal, if by "normal" is understood "average." He had finished fourth grade at the age of ten years and three months, the proper age for a child who enters first grade at the age of six. His I. Q. was 94 - a mere point below that which would place him in what is commonly designated the normal group. His scores on both the initial and final tests in arithmetic gave him a grade of 5-0 -- again, a perfectly normal or average score.

Obviously, this perfectly "normal" group of children can never be expected to do brilliant work, and the best that can be hoped for is reasonably satisfactory work if conditions are favorable. In George's case conditions were not favorable.

His chief handicaps were:

1. Poor physical condition.
2. Foreign-language speaking home.
3. Lack of inspiration, with a resultant satisfaction with mediocre performance.

To the teacher who finds in her class a student of George's type the following suggestions may be helpful. Encourage the parents to have the physical defects of the student removed or remedied as speedily as possible. Give the child many opportunities for oral recitations. These opportunities may come in the recital of poems, in the participation in socialized recitations, in oral reading, and in oral compositions. Development of effective study habits will produce results of better quality in the content subjects. The ability to handle reading of the work-type may develop power in the oral presentation of factual matter and in the solution of arithmetic problems.

For a short period, work in arithmetic slightly below the child's power might be offered so that he could have the experience of successfully completing assignments. Following this, in work of his own grade level, low marks might provoke dissatisfaction and irritate him to the point of checking his answers and keeping his errors to a minimum. Problems in which inaccurate results would bring annoyance to his classmates and to himself should be presented consistently. In this type would come the class projects of making articles, handling change,

measuring, etc.

If the child has any accomplishments such as violin or piano playing his appearance on school programs would perhaps counteract the feeling of inferiority which the weakness in the academic subjects might promote. After an attitude of self-appraisal in academic subjects is established, friendly competition with students near his level of ability could be encouraged in the effort to bring him to the higher goal of striving to surpass his own previous best record.

Case Study of S.A.D.

Shirley entered summer school at the age of nine years and eight months. She had finished fourth grade. The week before the term opened she had her tonsils removed. The removal of the tonsils seemed to be the only needed correction to render her physically fit. For this reason no physical examination at the school was given.

The scores in preliminary test in arithmetic were so low that neither her educational age nor her grade equivalent could be estimated in any one of the processes. The total score showed her grade equivalent to be 4.2, but again the same score was too low to estimate her educational age. It was considered inadvisable to interpolate the scores of this test. The first form of the reading test was not given because of the absence due to the tonsilectomy.

The teacher was amazed at finding that Shirley's I.Q. was 120. This meant that she had the highest I. Q. of any child in the group. She was a docile, timid child. Even in the third week of the term she would ask permission to get a pencil although she had been told many times to help herself whenever necessary. The teacher felt a barrier which was never worn down between the child and herself all through the summer period.

Shirley did not mix with the other children of the group. There was a lack of sympathy and co-operation between the home

A request to have eight multiplication combinations heard at home was returned with the statement that no one had time to do this and that the teacher was to do all she could for Shirley.

Her work was untidy and very careless. She lacked initiative. She did not care for school but was tolerant of it. Even at the end of the term she had to be reminded to check her work. Her attention span was woefully limited. This may account for the fact that she reached a low fifth-grade level in multiplication and was still ungradable in addition. The short addition required in multiplication required concentration of less length than that of column addition.

The child was entirely upset by any change of procedure. There was no apparent transfer in the ability to handle identical elements in the various forms of a process. A sample of this occurred in the subtraction process. After a fair degree of power in the subtraction combinations had been attained the child was unable to supply the missing minuends or subtrahends when the examples were written in the following form:

$$\begin{array}{r} 6 \\ - \\ \hline \end{array} \qquad \begin{array}{r} - \\ 9 \\ \hline \end{array}$$

Shirley's third and fourth years had been poorly attended because of difficulties arising from the diseased tonsils. This explains her ignorance of the division forms, long or short. If her parents had been understanding, the wisest procedure would have been to have her repeat the fourth grade. The "lost"

year could easily have been made up before she finished eighth grade.

In the report to her parents the school officials informed them that the lack of correlation between Shirley's mental power and her achievement was a matter of astonishment to the faculty. Another cause of wonder was the lack of consistency between her careless, unchecked work and her obedience to and tolerance of orders. It was suggested that a psychiatrist might reveal some emotional disorder or that a social worker might assist in a home adjustment that would help conditions.

A study of the achievement and the error charts show much improvement during the work of the term. This development in a child with an I. Q. of 120 should have been of greater extent considering the ideal conditions offered for work.

The writer discovered no error that was outstanding during the term; no set of number combinations that caused the student special difficulty. The attack had to be general, as the weaknesses were so general. To summarize: the teacher's work resolved itself into the development of study habits, attitudes, and the automatization of combinations of the four fundamentals.

In another portion of this paper will be discussed the interference which occasionally occurred in the mental processes.

In the Compass Survey Test, Advanced Examination: Form A Shirley attempted seven addition examples. Two were computed accurately. Three of those attempted were beyond the grade work

she had experienced. Two of the remaining four were incorrect because of errors in adding a "seen" to a "thought of" number.

In the subtraction part of the test mentioned above the student attempted five and correctly finished two examples. The three errors were due to ignorance of the subtraction combinations.

Five examples were attempted in multiplication. One was correctly computed. Two of the errors were concerned with problems beyond Shirley's grade. Two examples were incorrect because of errors in the multiplication combinations.

Six division examples attempted resulted in one correctly completed. Three of those incorrectly worked showed a lack of knowledge of the long division process. One showed ignorance of the method of dividing denominate numbers. In another incorrect example the difficulty arose because of inability to handle the division combinations. In all processes the student displayed no power to place the decimal point.

In general problems, one of two attempts was incorrect. As the answer was not accompanied by any computation the writer was unable to locate the source of difficulty.

With the completion of the preliminary test the teacher realized that drill work in the combinations of four fundamentals was imperative. Three of the first five errors checked involved difficulty with the subtraction, multiplication, and division combinations.

The Compass Survey Test was followed by two diagnostic tests. The Compass Diagnostic Tests were used to locate the student's weaknesses in division and in addition.

In the basic addition facts thirty-four of thirty-five attempts were correct. The successful attempts included twelve combinations of the first, six combinations of the second, eleven combinations of the third, and six combinations of the fourth quadrant.

In the higher decade addition seven errors occurred in fifty-four attempts. Forgetting to add the carried number resulted in the wrong answer four times. Errors in three addition combinations produced three incorrect sums.

Nine of ten examples in single column addition were completed successfully. The error committed was that of failure in adding a "seen" to a carried number.

In column addition with carrying a grade of 100 per cent was reached on nine attempts.

Six efforts to check addition examples by adding downwards resulted in a mark of zero. It is difficult to reconcile the grade of 100 in column addition above with a grade of zero in checking the same type of material. The writer offers two probable causes for the difference in grade: Shirley was accustomed to upward adding, or the change in process may have affected her. The appearance of the wrong answer may also have influenced the accuracy of her computing.

TABLE III

Number of Problems Attempted by Shirley, Number Right;
and Percentage of Accuracy

	Addition			Subtraction			Multiplication			Division			Problems		
Preliminary	150	127	84.6	5	2	40	5	1	20	6	1	16.6	2	1	50
							64	49	92.1	5	4	80			
										20	1	5			
										22	8	44.4			
27	13	10	76.9				6	4	66.6						
28	12	10	83.3	24	21	87.5									
29				6	6	100									
30							21	18	85.5						
1							3	1	33.3						
5	23	23	100	25	24	96							5	5	100
6				22	22	100	4	3	95						
7	12	12	100				15	13	86.6				5	5	100
8	18	16	88.8	5	5	100	7	0	0				8	8	100
11	112	112	100				19	19	100				17	14	82.3
12	112	109	97.3	112	111	97.1							3	3	100
13	80	77	96.2				24	19	79.1				13	13	100
14				80	77	96.2							13	10	76.9
15	43	42	97.6				31	31	100				7	7	100
18							8	1	12.5				10	8	80
19	24	24	100				20	19	9.5				10	6	60
20							6	5	83.3	10	6	60	10	9	90
21							5	5	100				6	1	76.6
22							5	5	100	10	7	70	10	4.5	45
25	2	2	100							10	8	80	8	7	87.5
26	2	2	100				5	2	40	4	4	100			
27										3	3	100	2	1	50
28	2	2	100	2	2	100	2	1	50						
	23	22	95.6	20	17	85	20	20	100	19	5	26.3			
Final	6	3	50	7	2	28.5	5	3	60	3	0	0	4	2	100

TABLE IV
 Opportunities of Shirley for Errors of 18 Types, by Days;
 Number of Errors Made; and Percentage of Accuracy

Error	1	2	3	4	5	6	7	8	9
Preliminary	2 26 76.1 5 23 1 2.1 3 130 2.3 2 132 1.5	1 - -	3 15 20 4 48 8.3	2 16 12.5 3 69 4.3 1 20 5 2 38 52	1 4 25 1 2 50	4 143 2.7	5 51 9.8	2 - -	18 20 90
6				1 20 5					
7			1 30 33.3	1 49 2.0					
8	2 76 2.6								
11									
12	1 112 .8		1 112 .8						
13				2 72 2.7		3 80 3.6	3 72 4.1		
14			3 80 3.7						
15						1 43 2.3			
18									
19				1 20 5					
20					1 26 3.8				
21									
22					1 26 3.8				
25				1 36 2.7					
26						2 27 7.4			
27									
28				1 14 7.1					
Final	1 39 2.5								

[illegible]

The diagnostic test in division disclosed the fact that the student was ungradable in any phase of the process except its vocabulary. In this function she made a grade of high-fourth. As had been indicated in the survey test, the student evidenced no knowledge of long division form. On division combinations from a possible twenty she answered one correctly.

In the multiplication which accompanied the division examples of this test fifty-nine of sixty-four combinations were answered perfectly. Twenty-six of twenty-nine combinations occurring in the dividing were correct. A score of zero on short division with carrying was obtained toward a possible twelve grade. In the estimation of the first quotient number a score of sixteen toward a possible seventy was earned.

Thirteen addition examples of increasing length and difficulty were attempted by Shirley. Ten were computed accurately, but too slowly. Shirley counted to obtain results. She had not developed automatic control of the combinations, especially those of higher decades. She never considered the checking of results. The same error occurred three times in this exercise; that of adding a "seen" to a "thought of" number.

Six multiplication examples with multiplicands and multipliers of two digits each received a grade of 66.6 per cent. Two were incorrect because of confusion of the processes. For the first partial product the student added the multiplier and multiplicand. Shirley had not acquired the habit of placing the

sign for the process she was using. This may have accounted for the two errors.

This paper will not present a daily description of the assignments, achievement, accuracy of achievement, error types, their frequency and rate of frequency.

1. Mistake in adding a "seven" to a "thought of" number.

$$\begin{array}{r} 967 \\ 298 \\ 573 \\ 764 \\ \hline 2612 \end{array}$$

2. Copied figures incorrectly.

3. Error in subtraction combinations.

$$\begin{array}{r} 124.63 \\ 14.98 \\ \hline 109.65 \end{array}$$

4. Error in multiplication combinations.

$$\begin{array}{r} 561 \\ \times 107 \\ \hline 4067 \\ 5610 \\ \hline 60167 \end{array}$$

5. Error in division combinations.

$$7 \overline{) 450} \begin{array}{l} 50 \\ 450 \end{array}$$

6. Error in addition combinations.

$$\begin{array}{r} 5 \\ 4 \\ \hline 13 \end{array}$$

7. Forgot to add carried number.

$$\begin{array}{r} 3 \\ 18 \\ \hline 11 \end{array}$$

8. Confusion of processes.

$$\begin{array}{r} 8 \\ \hline 68 \\ \times 13 \\ \hline 81 \\ 681 \\ \hline 761 \end{array}$$

9. Intentional omissions.

10. Forgot to add the carried number in multiplying.

$$\begin{array}{r} 27 \\ \times 6 \\ \hline 142 \end{array}$$

11. Reversal of digits.

31 for 13.

12. Forgot that she had borrowed.

$$\begin{array}{r} 985 \\ 258 \\ \hline 637 \end{array}$$

13. Carried the wrong number.

$$\begin{array}{r} 834 \\ \times 7 \\ \hline 5938 \end{array}$$

14. Ignoring the remainder after estimating quotient number.

$$\begin{array}{r} 14 \\ 8 \overline{) 8} \\ \hline 65 \end{array}$$

15. Ignoring zero in dividend.

$$\begin{array}{r} 27 \\ 3 \overline{) 810} \end{array}$$

16. Failure to place a zero in the quotient.

$$\begin{array}{r} 37 \\ 3 \overline{) 921} \end{array}$$

17. Wrong remainder after proper estimate of quotient.

$$\begin{array}{r} 87-4 \\ 4 \overline{)351} \end{array}$$

18. Did not place partial product properly.

$$\begin{array}{r} 1522 \\ \times 290 \\ \hline 136980 \\ 3044 \\ \hline 167420 \end{array}$$

June 28

Ten of twelve examples with three four digit columns each were worked correctly. The error of the previous day occurred twice. The student never wrote the number which was to be carried. With a child whose attention span was so limited the development of this habit would have been an aid to accuracy.

Twenty-one of twenty-four subtraction examples were accurately completed. The four mistakes were of the same type. They were due to lack of knowledge of the subtraction combinations.

The tables of 6's were written perfectly. The oral work involved flash card drill and correction of errors of the previous day's work. The teacher encouraged the checking habit, the habit of writing down the carried number, and the endeavor to write more legibly.

June 29

Six of six subtraction examples were graded 100 per cent.

Nine of ten multiplication examples with one digit multipliers were right. The error occurred in the multiplication combinations.

The tables of 7's and 8's were written perfectly. In the flash card drill intensive work was done with 7 and 8 as multipliers.

June 30

Shirley completed correctly sixteen multiplication examples with one-digit multipliers. Three attempts contained errors. One error was due to failure to add the carried number. Mistake in multiplication combinations were responsible for the other two inaccuracies.

She was then given three multiplication examples with two-digit multipliers. Only one was accurately completed. The first error was the confusion of processes mentioned earlier in this paper. The second error was a reversal of digits.

An exercise involving money-writing was unsuccessful. Two of twelve money amounts were properly written. As these were the first two a knowledge of the process was apparent. Fatigue, limited attention span, or both of these may have produced this disappointing result.

July 1

Most of this morning was devoted to oral drill, individual flash card drill, and correction of the work of the previous day.

One of three multiplication examples was worked correctly. Two failures to add the carried number produced the inaccuracies. Part of the paper was filled with marks which showed that the student dawdled a portion of the period.

At the end of the first week the teacher's new concerns were the student's timidity, slowness, short attention span, and aversion to checking.

July 5

Shirley correctly worked twenty-four of twenty-five subtraction examples. Seven of these involved borrowing. Twenty-three single column additions were perfectly finished. Five of five simple one-step problems were solved accurately. The teacher had planned problem material several days before this. The slowness of the student's computing retarded matters. Problems had been solved orally several days previous to this.

July 6

The student worked twenty-two of twenty-two subtraction examples correctly. Three of four multiplication examples with two-digit multipliers were right. The error was one in the multiplication combinations.

July 7

Thirteen of fifteen multiplication examples with one-digit multipliers were correctly finished. An error in the multiplications, and the carrying of a wrong number were the inaccuracies.

Twelve simple addition examples involving the higher decades were completed without an error.

Seven of nine examples in subtraction was accurately completed. One error occurred in the subtraction combinations. The other was that of forgetting she had borrowed. The paper was very untidy and carelessly arranged. This was a surprise, as the work for a few days previous had been neatly arranged and legibly written.

Five simple problems involving money-writing earned a 100 per cent grade.

July 8

Five multiplication examples with multipliers of 6, 7, 8, 9, and 3, respectively, were perfectly finished.

Eight problems in the subtraction process were done correctly.

This was the first day Shirley had been assigned division work. Seven attempts were incorrect. Shirley was able to estimate the first quotient number, but she had no notion of how to handle the remainders.

Sixteen of eighteen single-digit columns were added accurately. The errors occurred in adding "seen" to "thought of" numbers.

The end of the second week found the student able to produce more work. She seemed less timid and more interested.

July 11

The third week the student used a work book. This increased the number of solutions reached, as much written work was avoided. In writing numbers, fifty-four of fifty-eight answers obtained were correct.

One hundred twelve of one hundred twelve addition combinations were perfect.

Fourteen of seventeen one-step problems involving simple addition combinations were right. In those incorrectly solved the computation was correct, but the wrong process had been selected.

Shirley correctly worked nineteen of the most simple short-division examples the teacher could provide. The divisor in each case was 2 and the dividends ranged from 2 to 20. The aim was to establish the short division form and the notion of a remainder. The teacher felt that she had offered the student work which was too difficult on the previous day.

July 12

To give the student an opportunity to follow directions and to provide a little variety, time-telling and counting exercises were given. She was successful in both exercises.

In a test of subtraction facts one hundred eleven of one hundred twelve were correct. The error was one in the subtraction combinations.

In a test of subtraction in which missing minuends and subtrahends were to be supplied the difference between the two given numbers was inserted as in the answer in almost every case.

$$\begin{array}{r} 0 \\ -7 \\ 7 \end{array} \quad \begin{array}{r} 0 \\ -8 \\ 8 \end{array} \quad \begin{array}{r} 1 \\ -8 \\ 9 \end{array}$$

The ability to follow directions needed to be encouraged. One hundred nine of one hundred twelve easy addition examples were accurately answered.

Three simple problems in addition were solved accurately.

July 13

In eighty addition examples the student worked seventy-seven correctly. The three errors occurred in the addition combinations.

Of eight problems in addition eight were correct.

Five of five problems in multiplication of money were correct.

Short division had been discussed, work of other children had been shown to Shirley, and flash card drill in the division and multiplication combinations had continued daily.

Nineteen of twenty-four examples in the multiplication of money were correct. The errors occurred in the failure to add the carried number, and in the combinations.

July 14

July 14

Ten of thirteen problems were accurately finished. Two errors were due to the selection of the wrong process and one was due to misunderstanding of the problem's wording.

Seventy-seven of eighty subtraction examples were correct. The three errors occurred in the combinations.

July 15

In the writing of money numbers a grade of ninety-three was made. Forty-two of forty-three simple addition examples were correct. An error in addition combinations was made.

Thirty-one simple short division examples with a divisor of 7 and quotients of one digit merited a 100 per cent grade. Shirley had such poor habits of study and so many weaknesses to improve that opportunities to attack the short division were not so plentiful as the teacher would have liked them to be.

Seven of seven problems were solved correctly.

July 18

Shirley was presented with short-division examples entailing two or more digits in the quotient and the remembering of a remainder. In two or three she handled the carrying nicely, and then spoiled the result by ignoring a zero in the dividend, or a zero in the quotient. Only one of ten attempts was correct.

Eight of ten problems were correctly finished. One was

intentionally omitted, and in the other the wrong process was selected.

July 19

Twenty-four of twenty-four multiplication combinations were correct.

Nineteen of twenty division combinations were perfect.

Six of ten problems were solved correctly. Two were wrong because the wrong process was selected, and two because of a subtraction and a multiplication error.

July 20

Five of six multiplication examples were correct. An error occurred in a multiplication combination.

Nine of ten problems were correctly solved. The incorrect one was incomplete.

Six of ten short division examples were correctly computed. The grade was not high, but it was a splendid step in the right direction. Shirley's attention was drawn to the differences in the results in dividing 48 and 408 by 2. The work involved carrying remainders, the divisors were varied and the dividends were three, four, and five digits. The student felt greatly encouraged.

July 21

Only one of six two-step problems were solved correctly.

Shirley did not seem to be able to carry through when work increased in difficulty.

Five multiplication examples with 5, 6, 7, 8, and 9 as multipliers earned a 100 per cent grade.

July 22

Shirley's papers began to deteriorate in tidiness and legibility. The writing was not even of second-grade level on some days. The instant the difficulty of the work increased a slump in good habits fairly well established was apparent.

Four and a half of ten two-step problems were correctly solved. Three were incomplete, one was intentionally omitted, one was incorrectly computed, and one was missed because of inadequate background. Shirley thought that six pints make a quart.

Five of five multiplication examples were correctly worked.

Eight of ten short division examples involving conditions mentioned on preceding days were correct. One error was made in the division combinations and another occurred in ignoring a zero in the quotient.

July 25

Seven of eight problems were finished perfectly. In the incorrect one Shirley chose the wrong process.

Shirley was given some long-division examples with divisors of 50, 20, 30, 40, 62, 62, 32, etc. Eight of ten were

right. Several of them had remainders in the answer. One error occurred in the selection of the wrong quotient number. The other error was one the writer is unable to classify except by saying that the first number of the dividend was ignored.

$$\begin{array}{r} 30 \overline{) 697} \\ \underline{60} \\ 637 \end{array}$$

This may have been a visual slip.

Two of two addition examples with three five-digit columns were right.

Three of four multiplication examples with two- and three-digit multipliers were correct.

July 26

On four long division examples with divisors of 40, 34, 50, and 63 Shirley made 100 per cent.

On two addition examples of fifth-grade difficulty Shirley graded 100 per cent.

On five multiplications no more difficult than those graded 100 two days before the student made only 40 per cent.

July 27

Most of the work on this day was oral.

Three of three long division examples were correctly worked.

One of two problems was correct.

July 28

In a mixed fundamental drill the following results were obtained:

Addition	100 per cent
Subtraction	100 per cent
Multiplication	50 per cent

In a graded test in the fundamentals the following results were obtained:

Addition	95.6 per cent
Subtraction	85.0 per cent
Multiplication	100 per cent
Division	26.3 per cent

July 29

The Form B of the examination given on entrance was administered. The record of both tests appears in the following paragraph.

Arithmetic Test Scores, Educational Age, and Grade Equivalent
of Shirley in Initial and Final Tests

Processes	Compass Survey Test Educational Age Grade Equiv.					
	Initial	Final	Initial	Final	Initial	Final
Addition	2	3	-	-	-	-
Subtraction	1	2	-	-	-	-
Multiplication	1	3	-	9-6	-	L 5
Division	1	-	-	-	-	-

Arithmetic Test Scores, Educational Age, and Grade
Equivalent of Shirley in Initial and Final Tests

Processes	Compass Survey Test		Educational Age		Grade Equivalent	
	Initial	Final	Initial	Final	Initial	Final
Percentage	0	0	9-0	-	-	-
Problems	1	2	-	-	-	H 4
Total	6	10	-	-	4.2	4.6

Reading Test Scores, Reading Age, and Grade
Equivalents of Shirley in Final Tests

(Shirley missed the initial test in reading)

	New Stanford Reading Test		Reading Age		School
					Grade
Paragraph Meaning	59		10-7		4.6
Word Meaning	56		10-5		4.4
Total (average) Reading	57.5		10-6		4.5

Conclusions

1. The student's attention span was very short.
2. The student's memory span was decidedly limited.
3. Any element of change in a familiar situation produced disastrous results.
4. Temporary lapses in apparently well-established habits occurred several times during the term.

5. Interference in the mental processes was obvious in several error types.

6. Bonds of learning in the fundamental processes and in their basic facts were weak.

7. The child was timid to the point of being suppressed.

8. The teacher suspected some emotional disorder which was not apparent to even an interested observer.

9. There was a mere endurance of school work which precluded any pleasure or enthusiasm in it.

10. General weakness rather than special difficulties characterized the student's work.

11. An I. Q. of 120 does not necessarily mean satisfactory school achievement.

Shirley belongs to that group of children whose achievement is far below that which their I. Q.'s would lead a teacher to expect. Interfering factors work against the achievement. These interfering factors may be excessive introspection, day dreaming, short attention span. In classrooms this type of pupil will be found sitting quietly and causing the teacher no annoyance in matter of discipline. For this reason she is apt to be overlooked, misunderstood, or classified as a child of mediocre or even inferior ability. She presents, because her weaknesses are so general, a much more difficult problem than does the student with weaknesses that are specific.

The teacher must not be deceived into forgetting this quiet, good child as she may be the one most in need of considerate attention. Often, as in this particular case, the student is a day dreamer, perhaps endeavoring to block off a part of her personality from the rest of self and reality. A feeling of inferiority coming from failure in a particular subject (this type of failure is usually so marked before classmates) may inspire the child to set off on a trip to her inner thoughts and to indulge in what we commonly call a day dream. Morgan tells us that excessive day dreaming shows a dissatisfaction with reality (29:103).

This type of pupil should have many opportunities for activities which entail relationships with other people. Such activities may occur in the delivering of messages, passing papers, heading the lines at recesses and dismissals, playing games with others, and so forth. The encouragement of all activities which demand oral expression might serve as an outlet for the suppression in the child's behavior. Encouragement of her own initiative is also an invaluable developing influence.

At all times the teacher must be the friend who is willing to help, yet careful not to force herself, lest her anxiety defeat her objective - to build up the child's confidence and to eliminate the feeling of fear. The teacher must be careful

not to confuse sympathy with sentiment. If she cannot obtain the confidence of the student she should try to find someone who can. Only a teacher with an inflated ego will feel that she can win every child's confidence.

The teacher should arrange the work so that the pupil tastes the joy of victory. When a certain degree of self-confidence is established she should shift the work to the field in which the weaknesses lie. A discreet and tactful talk to the child concerning the weaknesses may be helpful. The endeavor to build up an attitude of facing the weaknesses and removing or modifying them is of great importance.

This type (to the writer) is one of the most difficult to reach. The teacher must remember that she is working upon something intangible and that the results of her work are far reaching. She may never know that she has succeeded. She may never know that she has produced a condition more pitiable than the one upon which she began her work.

Case Study of M. M.

Margaret had finished sixth grade when she came to summer school. She had an I. Q. of 101. Her age was twelve years and eight months. She was nervous, temperamental, hypersensitive, and had the disposition of a persistent worrier. She was exceedingly anxious to progress, pleased with praise, and deeply downcast over adverse criticism.

At first her tenseness about the work bordered on hysteria. She wanted to work during recess even on the warmest days of July, 1932. She could scarcely wait till the teacher finished with the other pupils before coming up for the next assignment. Much of this drive was within herself, but there was a motivation from her mother which was too powerful for the child's good. She would ask to take home extra work to do at night. For half an hour before school opened each morning Margaret had a private tutor in arithmetic.

The student enjoyed talking about herself. During the physical examination she admitted that she had had fainting spells, and that she had once been under emotional strain. The doctor reports that she said "sets into tantrums of fear - is that convulsions?" Two years before she had a nervous breakdown which removed her from school for nine months. The desire to regain the lost time may have urged her to work slavishly, although she never discussed this loss of time with the teach-

er. The doctor considered her normal physically.

A splendid gain was made during the five-week period in the fundamentals, problem-solving, paragraph comprehension, and word selection. A two-year gain in reading showed that power in reading problems had transferred to the ordinary reading content. A three-year growth in addition and division was shown in a comparison of the initial and final tests.

In the preliminary test the student was ungradable in addition. Her trouble occurred in single-column addition and downward checking of three-and-four column addition. In the same test she was also ungradable in general problem work. In the Compass Diagnostic Test XIX: General Problem Scale she made an equivalent to Low Sixth. In the Final test her rating was a grade equivalent to Low Seventh. The total gain for the five weeks was a grade year from 5.8 to grade 6.8. This is the highest gain recorded among these 6 case studies.

This student ranked next to the lowest in regard to her I.Q. The teacher's part in the term work was to develop the habit of checking, promote the ability to analyze a problem into its elements, and to help the little girl to relax.

Below follows a daily description of the error types, error samples, accomplishment, accuracy of achievement, the error and accuracy charts.

TABLE V

Number of Problems Attempted by Margaret; Number Right;
and Percentage of Accuracy

	Addition			Subtraction			Multiplication			Division			Problems			Decimal Notation		
Preliminary	7	3	42.8	9	6	66.6	10	5	50	6	4	66.6	7	1	14.2			
Diagnostic	23	23	100	8	8	100										10	9	90
	5	4	80													10	10	100
	7	5	71.4													15	7	46
28	10	8	80															
29							10	10	100				9	9	100	9	5	55.5
30	1	0	0										17	8	47			
1	1	1	100													8	7	87.5
5	10	10	100	5	5	100							8	8	100			
6	5	4	80															
7													12	12	100			
8													5	5	100			
11	26	25	96.1										5	3	60			
12				32	28	87.5	12	6	50				2	1	50			
13							9	6	66.6									
14							14	14	100									
							7	1	14.2									
15							5	1	20				11	11	100			
18							4	4	100	4	2	50						
19	3	3	100	1	1	100	1	1	100	1	1	100						
20	42	42	100							30	22	73.3						
										5	4	80						
21	24	24	100							5	5	100						
										3	3	33.3						
22	4	4	100	2	2	100	2	2	100	2	2	100	2	2	100			
25	2	2	100	2	2	100	2	2	100	2	1	50	3	2	66.6			
26				42	41	97							5	4	100			
27	2	1	50	2	2	100	2	1	50	2	1	50	5	4	80			
							2	2	100	2	2	100	10	8	80			
28	29	29	100	30	29	96	2	0	0	2	2	100	1	1	100			
Final	10	8	80	9	5	55.5	8	4	50	25	24	96	6	4	66.6			

TABLE VI

Opportunities of Margaret for Errors of 16 Types; by Days; Number
of Errors Made; and Percentage of Error

	1	2	3	4	5	6	7	8
27	2 46 43	2 - -	1 7 14.2 1 3 33.3	1 1 100	1 2 50 1 28 3.5	1 2 50	1 1 100	1 1 100
28	2 93 2.1	2 - -						2 6 33.3
29								
30	2 12 16.6				4 40 10			1 15 6.6
1	2 122 6.6							
6	1 52 1.9							
8	1 23 4.3							
11	1 228 .4							
12								1 6 16.6
13								
14								1 7 14.2
15								
18					1 17 5.8			
19			5 29 3.4		1 29 3.4			
25	1 8 12.5 1 27 3.7							1 2 50
26								
27			3		1			1

TABLE VI (continued)

	9	10	11	12	13	14	15	16
27	1 2 50							
28								
29		4 9 44.4						
30		1 15 6.6	1 6 16.6	1 2 50				
1				1 8 12.5				
6					1 8 12.5			
8								
11			2 127 1.5			2 70 28		
12					1 26 3.8		1 19 5.2	1 4 25
13							2 35 5.7	1 1 100
14							1 32 3.1	1 1 100
15					2 33 8.6		2 26 7.6	
18								
19								
25								
26			1 66 1.5		1 10 10		1 16 6.2	
27			1					

1. Error in adding a "seen" to a "thought of" number.

$$\begin{array}{r} 967 \\ 298 \\ 573 \\ 764 \\ \hline 3302 \end{array}$$

2. Error in copying.
3. Confusion in processes.

$$\begin{array}{r} 9 \quad 2 \quad 9 \\ 1\frac{4}{7} \quad \frac{2}{7} \quad 4\frac{9}{7} \\ 7 \end{array}$$

4. Ignorance of borrowing procedure in the subtraction of mixed numbers from whole numbers.

$$\begin{array}{r} 12 \\ 7-2 \\ \hline 4-2 \\ \hline \end{array}$$

5. Error in multiplication combinations.

$$\begin{array}{l} 9 \times 6 = 56 \\ 9 \times 0 = 9 \end{array}$$

6. Inability to multiply a fraction by a whole number.

$$\frac{2}{3} \times 10 = 6$$

7. Inability to multiply denominate numbers.

$$\begin{array}{r} 9\text{yd } 2\text{ft} \\ 7 \\ \hline 64\text{yd } 4\text{ft} \end{array}$$

8. Omission of decimal point.

$$\begin{array}{r} 85 \\ 21 \overline{17.85} \\ 168 \\ \hline 105 \\ \hline 105 \end{array}$$

9. Failure to reduce to lowest terms.

$$\frac{23}{8} \times \frac{9}{5} = \frac{115}{72}$$

10. Error in placement of decimal point.

$$\begin{array}{r} 21 \quad 418 \\ \underline{.19-19} \\ 21 \end{array}$$

11. Mistake in subtraction combinations.

$$\begin{array}{r} 98.0 \\ -15.7 \\ \hline 82.2 \end{array}$$

12. Selection of wrong remainder after correct estimation of quotient digit.

$$\frac{7}{8} = 8 \quad \begin{array}{r} 7 \\ 7.00 \\ \underline{88-7} \\ 8 \end{array}$$

13. Forgot to add carried number.

$$\begin{array}{r} 17 \\ 16 \\ 3 \\ 398 \\ 184 \\ \hline 618 \end{array}$$

14. Forgot that she had borrowed.

$$\begin{array}{r} \$8.35 \\ 7.57 \\ \hline .88 \end{array}$$

15. Carried wrong number in multiplying.

$$\begin{array}{r} 15.5 \\ 9.3 \\ \hline 465 \\ 1185 \\ \hline 12415 \end{array}$$

16. Error in placing partial product.

$$\begin{array}{r}
 86,7 \\
 64.0 \\
 \hline
 34680 \\
 5202 \\
 \hline
 86700
 \end{array}$$

In the addition part of this test seven were attempted and three were correct. Three types of errors were discovered. Two mistakes were made in adding a "seen" to a "thought of" number. One error was due to copying the wrong number. The last error was that of confusing the multiplication process with that of addition.

In subtraction, a rating of $66\frac{2}{3}$ per cent in accuracy of accomplishment was attained. Six of nine attempts were correct. Two errors occurred in the subtraction of mixed numbers. Through the first error the student displayed no knowledge of the borrowing procedure when a mixed number was subtracted from a whole number. The second error was that of multiplying instead of subtracting. In the third example a change of common denominators was necessary. Correct numerators were found, but the student used 56 instead of 54 as the denominator. This error was recorded as ignorance of the multiplication combination.

Five of ten attempts in multiplication were correct. The first mistake was in the multiplication combinations. The second error was that of copying the wrong number. The third and fourth errors showed lack of ability to multiply a whole

number by a fraction. The last error occurred in the multiplying of denominate numbers. No knowledge of the procedure was apparent.

In division four of six attempts were correct. The omission of a decimal point and the failure to reduce to lowest terms produced two inaccuracies.

One of ten attempts was correct in percentage. The teacher believed that this accuracy was one of chance. Most of the answers were guesses; and as Margaret's grade would indicate no experience in percentage, it is not being considered in this report.

In the problem work one of seven attempts was correct. Five of the attempts were beyond Margaret's grade experience. The last error occurred in the name of the answer. Linear instead of square units were named.

June 27

In the addition part of Buswell and John's Diagnostic Chart for Individual Difficulties in Fundamentals of Arithmetic Margaret made no errors in twenty-three examples. She discovered two errors in her checking and eliminated them before the grading of the paper.

Eight of eight simple subtraction examples were worked correctly. Work in the notation of decimals was offered to the student. Nine of ten were written correctly. The one

error occurred in the copying of the number.

In changing fractions and mixed numbers to decimal form, and the reverse, seven of fifteen were answered correctly. After an interview with the teacher and a checking of the work, seven of the eight incorrect answers were corrected.

Four of seven attempts in the addition of decimals were correct at the first effort. A check remedied one error before the grading of the paper. Two errors included the omission of the decimal point and a mistake in copying a number.

Margaret was given an exercise in finding errors in pointed answers. She graded 100 per cent on ten such problems.

Four of five examples in addition of irregular columns received a mark of eighty. The error was due to the incorrect copying of the problem.

An oral and written drill on aliquot parts and on changing fractions to decimals was given. This completed the morning's work.

June 28

Eight of ten addition examples of increasing difficulty were correct. A checking of the two incorrect examples remedied the errors.

June 29

Nine of nine money-writing problems were completed correctly.

In the making of a bill there were four errors in decimal-point location.

Ten of ten examples involving the multiplying of money were accurately finished.

June 30

In the making of a bill all items were listed correctly in regard to the location of the decimal point. The column writing was very irregular and an error occurred in the addition of the bill. The student recopied the column in a neat fashion, but the same type of error was committed again.

Margaret was given a set of problems and told to work as many as she could. She worked seventeen. Eight of these were accurately solved. Most of the errors occurred in computation. One problem was incomplete. This paper will be concerned with just the computation errors of the problems. Errors of reasoning were discussed with Margaret, but the writer is not recording them at this time.

There were four errors in the multiplication combinations and one in the subtraction combinations. Two errors occurred in adding a "seen" to a "thought of" number. One decimal point was misplaced; another decimal point was omitted. The wrong remainder was obtained after the selection of the correct quotient digit.

July 1

Seven of eight examples in changing fractions to decimals were correct. The error which occurred was that of choosing the wrong remainder after the correct quotient digit had been selected.

A bill was itemized correctly. It is listed in this paper as a problem in addition.

July 5

Five of five examples in subtraction involving borrowing were accurately finished.

Ten of ten addition problems of 3 four-digit columns received a 100 per cent grade.

Eight of eight two- and three-step problems were accurately solved.

Three problems not recorded in the accuracy of accomplishment chart were incorrect because of confusion of methods of finding area and perimeter and confusion of multiplication and division of fractions, shown by inversion in multiplication of fractions.

July 6

Four of five reasonable difficult examples in addition were correct. The student forgot to add the carried number.

In a second addition exercise four of five were correct. The error occurred in adding a "seen" to a "thought of" number.

Three of four problems were correctly worked. The incorrect one resulted from confusion of area with perimeter. Two of the correctly worked problems involved the writing of bills. A great improvement was noted in the regularity of money columns, arrangement of items, and the manipulation of the numbers.

July 7

Twelve problem attempts were accurately completed. The computation related to these problems involved addition and multiplication of fractions, decimals, and whole numbers.

July 8

In two sets of exercises the student wrote just the answers. Three errors in these answers could not be analyzed.

Five of five simple problem attempts were finished perfectly.

In the making of a bill an error in addition occurred. The failure to add the "seen" to the "thought of" error was one of the student's chronic ailments.

Three of five problem attempts were correct. The two errors lay in the confusion of method of obtaining perimeter with that of obtaining area.

Margaret was exceedingly anxious to progress. She was industrious to the point of hysteria. She delighted in good marks. She had the manners of a finished adult. When adverse

criticism was offered she felt it very keenly. She was very easily crushed. A mark below 85 was a distinct blow to her pride.

July 11

The third week Margaret was given a work book. In a test of adding three-figure numbers and larger numbers twenty-five out of twenty-six attempts were correct. The error of adding a "seen" to a "thought of" number occurred.

In an exercise in writing numbers, thirteen of twenty-two attempts were correct. Ten of the examples concerned Roman numerals. The student had had little experience in these.

Nine examples were given in the changing of mixed numbers to improper fractions. Seven were correct. The two errors may have occurred in higher decade addition or in the multiplication combinations. As the answer was not accompanied by any work, the marker could not decide on the type of error.

Samples follow:

$$3 \frac{7}{12} = \frac{42}{12}$$

$$14 \frac{5}{8} = \frac{109}{8}$$

July 12

In itemizing a bill a mistake in addition occurred. It was Error 1, referred to many times before in this paper.

Two attempts in keeping a cash account produced one cor-

rect answer.

In a subtraction test of thirty-four examples thirty-two attempts were correct. Two errors were in the subtraction combinations. A new error appeared. The student forgot that she had borrowed.

The student was given work in areas of rectangles, parallelograms, and triangles. The methods of finding these were reviewed before she began. Six of twelve were correctly worked. The wrong process was chosen twice. The other errors involved the carrying of the wrong number, omission of the decimal point, forgetting to add the carried number, and misplaced of second partial product when the multiplier contained a zero. These were considered examples in multiplication on the chart.

July 13

Fourteen of fourteen multiplication examples were correct. In an exercise involving the multiplication of money six of nine attempts were correct. The errors concerned the placing of the partial product and the carrying of the wrong number in multiplying.

July 14

One of seven examples in the multiplication of decimals was correct.

Three combination errors, the omission of decimal points,

and the incorrect placing of a partial product produced this low rate of accuracy.

This was the hottest weather of the summer of 1932. The weather may have lowered the student's power.

July 15

Eleven of eleven problems were accurately solved.

One of five examples in multiplication was correct.

July 18

Margaret came to school without her glasses. She seemed very tired and said she had been staying up late working on her arithmetic. She had not been sleeping well. The teacher told her that she was taking the work too seriously; that she should play at recess with the other children. She was urged to go swimming, riding, and so forth, and to stop worrying.

Seven of thirteen problems in mensuration were worked correctly. One omission and one error in copying were due to the forgotten glasses. Two problems were incomplete and in one the wrong process had been selected. No errors that could be termed computational occurred in the assignment.

Four of four examples in the multiplication of decimals received a grade of 100.

Two of four examples in the division of decimals were correct. One error was in a multiplication combination. The other occurred in the selection of the wrong quotient digit.

July 19

In a test in mixed fundamentals in whole numbers six of six attempts were correctly finished.

Four of four problems were accurately solved.

In the division of whole numbers and fractions by fractions and whole numbers twenty-two of thirty were correct.

Four errors occurred in a confusing of multiplication with division. One error was in a multiplication combination. Three could not be classified. A manipulation of figures had taken place.

July 20

In a test of reading numbers the student made a grade of 100.

In the addition of whole numbers forty-two of forty-two attempts were correct.

Thirty-seven of thirty-eight attempts in writing numbers were perfect.

Four of five examples in long division of decimals were correct. The error was in the multiplication combinations.

Nine of ten problems were solved accurately. Half credit was given to each of two different examples because they were incomplete.

Three of three long divisions were correctly computed, but

in two the decimal point was omitted of misplaced.

In division of fractions five of five attempts were correct.

In addition of longer columns twenty-four of twenty-four attempts were correct.

July 22

In subtraction of decimals two of two attempts were correct.

In addition of fractions four of four attempts were correct.

Two of two efforts in multiplication of decimals were accurately finished.

Two problems were completed perfectly.

Two of two attempts in division of decimals were correct.

In multiplication of a money amount by a mixed number the computing was correct, but the product of fraction and multiplicand was put in the wrong place.

July 25

Margaret's power to handle several processes during the period seemed to be growing.

Two of two attempts in multiplication examples in decimals were correct.

One of two examples in division of fractions was accurately completed.

Two of two attempts in multiplication of fractions merited 100 per cent.

One of two efforts in division of decimals was right. The error was due to omission of the decimal point.

Two attempts of two examples in addition of mixed numbers earned a 100 per cent accuracy grade.

Two of two examples in the subtraction of mixed numbers were right.

Two of three problems were correctly solved. The error was in computation. It was error one which has been recorded several times before.

Two of three examples in addition were correct. Error one was the offender again.

July 26

Four of five problems were correctly finished. The fifth was computed accurately but the wrong process was selected.

Forty-one of forty-two examples in simple subtraction were 100 per cent. The teacher believes that the error was due to visual weakness. This is the example:

$$\begin{array}{r} 18 \\ -4 \\ \hline 04 \end{array}$$

July 27

Four of five problems were correct. The student wrote thirty-five instead of $\frac{35}{2}$ for the answer. She overlooked the 2

One of two examples in multiplication of decimals was right. A mistake was made in the carrying of a number.

In two long-division examples with decimals one was correct. The error occurred in the placing of the decimal point. Margaret's tutor worked with her half an hour every morning. Either with her tutor or in day school Margaret had acquired a system of locating the point in division. The marker did not like to upset this habit as it had been fairly well established although not particularly effective.

In division, multiplication and subtraction of mixed numbers two of two attempts were successful. In addition of mixed numbers the fractions were handled perfectly and the whole numbers were forgotten.

Three of four problems were solved. The error was in the multiplication combinations.

July 28

Two of two problems in multiplication of decimals were wrong. Both were wrong because of mistakes in carrying.

Two of two examples in division of mixed numbers by mixed numbers two were attempted. One was correct and the other was incomplete.

Eight of ten problems were worked correctly. In one incorrect problem the numbers were so carelessly written that they could not be handled. In the second problem the wrong

process was selected.

In a test of addition of whole numbers twenty-nine of twenty-nine attempts were right.

In a test of subtraction of whole numbers twenty-nine of thirty attempts were right. The error was in the subtraction combinations.

In a test of multiplication of whole numbers twenty-seven of twenty-nine were right. The same error in multiplication combinations occurred twice.

In a test of multiplication of whole numbers twenty-four of twenty-five were right. A new error appeared in this test. A zero in the dividend was ignored.

July 29

In the final test eight of ten attempts in addition were correct. A new error caused trouble. The student carried unnecessarily. The other incorrect one was in the adding of denominate numbers. No help had been given by the teacher on this type of work during the term.

In subtraction five of nine attempts were correct. Three errors were due to confusion of processes; one was due to a mistake in the subtraction combinations. It is worthy of note that these same error types appeared in the preliminary examination.

Four of eight attempts in multiplication were accurately

worked. The same combination that was missed in the preliminary examination was missed in the final: 9×0 . The omission of the decimal point caused another example failure. One example involved the multiplying of denominate numbers. No work of this type had been given during the summer. The failure in this was easily understood.

In division seven of eight attempts were correct. The incorrect one involved the placing of decimal points. In this the student was unsuccessful.

In general problems four of six attempts were correct. One error was as follows:

$$\begin{array}{r} 14 \\ 14 \\ \hline 56 \\ 56 \\ \hline 616 \end{array}$$

The other was caused by lack of understanding of the word average.

A record of Margaret's standing in the initial and final tests in arithmetic and reading follows. The blank spaces indicate that her scores could not be interpreted in terms of educational age or grade equivalent.

Arithmetic Test Scores, Educational Age, and Grade
Equivalents of Margaret in Initial and Final Tests

Process	Compass Survey Test		Educational Age		Grade Equiv.	
	Initial	Final	Initial	Final	Initial	Final
Addition	3	8	-	12-0	-	H 8
Subtraction	5	5	10-0	10-0	H 5	H 5
Multiplication	5	5	11-6	11-6	H 5	H 5
Division	4	7	10-6	13-0	H 5	H 8
Percentage	1	0	-	-	-	-
Problems	1	5	-	12-6	-	L 7
Total	20	29	10	11-6	5.8	6.8

Reading Test Scores, Reading Age, and Grade
Equivalents of Margaret in Initial and Final Tests

	New Stanford Reading Test		Reading Age		School Grade	
	Initial	Final	Initial	Final	Initial	Final
Paragraph Meaning	44	73	9-6	11-9	3.7	5.9
Word Meaning	48	72	9-10	11-8	4.0	5.8
Total (average) Reading	46	72.5	9-8	11-8.5	3.9	5.85

The student's parents were informed about the progress made during the summer period. They were told that future progress depended upon Margaret's lengthened span of attention,

her inclination to check her work, the elimination of her excessive self-analysis, and her ability to accept criticism more impersonally.

From this case study the writer feels justified in making the following conclusions:

1. Unusual achievement may be accomplished through perseverance and persistence in spite of an I. Q. that is only normal.

2. Motivation which the teacher devised was as effective as the student's inner drive.

3. Excessive drill on a process or a combination later may result in interference which causes errors. The confusion of processes mentioned many times in the account of the student's accomplishment seems to exemplify this.

4. The short attention span was evidenced in the accuracy of the first and middle portion of an exercise and a trailing to inaccuracies at the end. Another sample of short attention span appeared in column addition. The errors usually occurred in the second or third columns of the example.

5. In her high rate of accuracy in exercises in mixed fundamentals the student's attention span showed development toward the end of the term.

6. A visual defect caused many of the errors found in arithmetic work.

Margaret belongs to that class of children whose tenacity, perseverance, and endeavor raise the quality and quantity of achievement to a higher level than the I. Q. would lead the teacher to expect. A comparatively modest endowment is supplemented by consistent plodding and a pronounced anxiety to succeed. These last mentioned helping factors are often utilized to the point of impairing health and peace of mind. The discovery of this type is one of the easiest tasks of the teacher. The first day an experienced teacher can select this worried type of pupil. He or she is very eager to have little talks with the teacher and to obtain extra privileges in the form of more attention and longer explanations.

The memory and the attention span is often limited and the student seeks to remedy these limitations by asking for more detailed explanations as mentioned above. The desire to attend and remember is evidenced in the demands for extra home assignments. In many cases, as with Margaret of this report, these repeated explanations and the completion of extra assignment plus a sincere effort to fulfill daily assignments produces results equal to and occasionally in excess of those obtained by students with higher I. Q.'s.

This type of pupil is often extremely sensitive to adverse criticism and to praise. The teacher is confronted with the many-sided task of changing the child's worried attitude to one of contentment, of teaching the child to meet defeat squarely,

of encouraging her to use past unpleasant experiences as a guide to preventing similar occurrences. The idea that no one expects more than one's best effort and this best effort modified by vital factors such as rest, health, recreation, and social contacts must be impressed upon the student. Care must be taken that the child realizes the happy middle ground of honest but not supreme effort.

Extra curricular activities should be encouraged. Interest in others should be developed. Visits to the parks, public places of interest, and schoolmates homes should be suggested. In short, let the pupil know that real education fits the individual to successful personal relations with his companions. The ease of discovering this type of student is succeeded by ease in realizing when improvement begins.

Case Study of Leo Nicholson

Leo had finished third grade when he entered summer school. His apparently good health rendered a physical examination unnecessary. His age was eight years and nine months. He had an I. Q. of 111. He was very slight in build and had an abundance of energy which he spend in fidgeting and consulting his neighbors. He had an appearance of alertness which was inconsistent with the superficial quality of his work. Leo was a willing boy; willing to the point of weakness; willing to work intensely or willing to play his time away. He needed steady guidance.

Preliminary tests revealed the facts that he was slightly below grade in paragraph and word meaning, and from six months to a year retarded in the fundamental processes. His attention span was very short; he dwadled and did not work to capacity. To gain time in "figuring out" a combination he would repeat the teacher's question three or four times. The multiplication combinations were not automatic.

Leo's mother wanted him to have remedial work in both reading and arithmetic, but the principal felt that one subject was all that Leo could manage during the short warm summer period and that this subject should be arithmetic. It was planned that Leo should have much problem work to increase his reading comprehension. To conquer his habit of repeating the

question, much oral drill was offered and he was requested to say nothing but the answer. The examples, very short and simple at first, were increased in length and quantity in an endeavor to increase Leo's attention span. Early in the term he was presented with a work book and after his oral drill and the completion of a set of daily assignments he was urged to work to capacity in this book. The beginning work in the book was slightly below Leo's grade and ability level, but he had the gratification of producing a large quantity of work with close to 100 per cent accuracy. It was also helpful in permitting an ease in computation while improvement in writing, paper organization, and industry was developed.

This brief, general sketch will be followed by a detailed day by day study of Leo's errors, their types, their frequency, the quantity and accuracy of his accomplishment in computation.

Error Types and Examples of Each Type

1. Confusion of processes.

$$\begin{array}{r} 8 \\ -5 \\ \hline 13 \end{array}$$

2. Mistake in addition combinations.

$$\begin{array}{r} 8 \quad 9 \\ 7 \quad 6 \\ \hline 14 \quad 14 \end{array}$$

3. Mistake in combining "seen" to "thought of" number.

$$\begin{array}{r}
 21 \\
 784 \\
 549 \\
 230 \\
 892 \\
 \hline
 2455
 \end{array}$$

4. Forgot that he had borrowed.

$$\begin{array}{r}
 50 \\
 -35 \\
 \hline
 25
 \end{array}$$

5. Borrowed unnecessarily.

$$\begin{array}{r}
 990 \\
 -628 \\
 \hline
 262
 \end{array}$$

6. Reversed minuend and subtrahend.

$$\begin{array}{r}
 53 \quad 35 \quad 82 \quad 86 \\
 27 \quad 29 \quad 26 \quad 19 \\
 \hline
 34 \quad 44 \quad 64 \quad 73
 \end{array}$$

7. Unintentional omission

$$204 \times 3$$

8. Error in multiplication combinations.

$$9 \times 4, \quad 4 \times 9, \quad 9 \times 8, \quad 8 \times 9$$

9. Intentional omission.

$$204 \times 6, \quad 174 \times 6, \quad 37 \times 8, \quad 8's \text{ in } 72.$$

10. Ignorance of division combinations.

$$14 \div 7, \quad 16 \div 3$$

11. Ignorance of carrying process in division.

$$\begin{array}{l}
 414 \div 9 = 40-4 \\
 627 \div 8 = 70-4
 \end{array}$$

12. Error in zero combinations.

$$0 \times 7, \quad 0 \times 5$$

13. Ignoring number in dividend when it was smaller than divisor.

$$648 \div 6 = 1-1$$

14. Carried wrong number in multiplying; carried the digit in the unit's place instead of the digit in the tens' place.

$$\begin{array}{r} 2 \\ 63 \\ 4 \\ \hline 262 \end{array}$$

15. Illegibility, (8 looks like 9).

$$\begin{array}{r} 68 \\ 19 \\ \hline 50 \end{array}$$

16. Incomplete because of lack of time.

$$\begin{array}{r} 1368-2 = 0 . . \\ 73 \\ \times 24 \\ \hline 292 \\ --6 \end{array}$$

17. Mistake in carrying in addition.

$$\begin{array}{r} 68 \quad 7 \quad 6 \\ 18 \quad 38 \quad 47 \\ \hline 76 \quad 55 \quad 63 \end{array}$$

18. Multiplied by carried number rather than by digit in multiplicand.

$$\begin{array}{r} 4132 \\ 48265 \\ 5 \\ \hline 245325 \end{array}$$

19. Mistake in obtaining remainder after selection of

proper quotient digit.

$$658 \div 5 = 131-2$$

20. Used quotient digit as divisor.

$$732 \div 6 = 126$$

21. Forgot to add carried number in multiplication.

$$\begin{array}{r} 56 \\ 467 \\ 9 \\ \hline 4243 \end{array}$$

22. Carried unnecessarily.

$$\begin{array}{r} 17 \\ 12 \\ \hline 39 \end{array}$$

23. Forgot to move partial product to the left.

$$\begin{array}{r} 36 \qquad 44 \\ 948 \qquad 278 \\ 39 \qquad 65 \\ \hline 7584 \qquad 1390 \\ 2844 \qquad 1668 \\ \hline 10248 \qquad 3058 \end{array}$$

In the accompanying chart the reader will see the frequency and order of these errors. Eighteen of these appeared fewer than seven times; five appeared only once, and five more appeared only twice. It is readily seen that Leo's greatest difficulties arose from lack of automatism in the multiplication combinations, (error 8), in the related division combinations, (error 10), and in the intentional omissions, (error 9), which resulted from this lack. A chart of error 8 is also included. This shows the opportunities for and commissions of error 8.

TABLE VII

Errors Made by Leo; by Days, by Frequency

Errors	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Preliminary			3	1	1	2		7	54	1		2		1			2			0	0		
27								2	2														
28								1	1														
29								1	1														
30								1	1	2													
1										2	2												
5							1	4															
6						4		6							1								
7								4	2				1		1								
8								2									2						
11										3													
12									1														
13		2																					
14	5							1									2						
15										1													
18								2	1	1													
19								1	2										1				
20	1								1	1							1						
21	1							3		2	2												
22								3		1													
25									2													1	2
26								2		3										1	1		
27																				1			2
Final			1							1						10							

TABLE VIII

Error 8: Mistakes in Multiplication Combinations
 Opportunities of Leo for Error 8 by Days; by Quadrants; Number of
 Errors Made, and Percentage of Accuracy

Quadrants	I			II			III			IV		
Preliminary	13	0	0	9	2	22.2	5	0	0	8	5	62.5
	13	0	0	20	0	0	16	0	0	21		
27	8	0	0	1	0	0	7	0	0	15	2	13.3
28				1	0	0	9	0		14	1	7.2
29							14	0	0	21	1	4.7
30							7	0	0	15	1	6.6
1												
5	1	0	0	11	0	0	8	0	0	17	4	23.5
6	6	0	0	16	0	0	6	5	83.3	8	1	12.5
7	7	0	0	6	0	0	14	3	21.4	26	1	3.8
8	3	0	0	10	0	0	6	1	16.6	20	1	5
11												
12												
13												
14	2	0	0	2	1	50	2	0	0	1	0	0
18	8	0	0				5	2	40	2	0	
19							1	1	100	1	0	0
20							11	0	0	17	0	0
21							8	1	12.5	12	2	16.6
22							8	1	12.5	12	2	16.6
25	2	0	0	4	0	0	8	0	0	8	0	0
26	2	0	0	3	0	0	8	0	0	8	2	25
27	3	0	0	10	0	0	4	0	0	3	0	0
28	29	0	0	14	0	0	29	0	0	27	0	0

TABLE IX

Error 8 Recorded in Quadrants for the Term

Preliminary Test

	4 x 8 4 x 9
	9 x 6 7 x 9 9 x 8 6 x 9 9 x 9

First Week

	9 x 8 6 x 8 8 x 6 6 x 9 6 x 9

Second Week

3 x 8, 4 x 9, 4 x 8 3 x 7, 4 x 9 3 x 7, 4 x 9 4 x 8 4 x 9	9 x 8, 6 x 9 8 x 8, 6 x 9 8 x 8 6 x 9

Third Week

3 x 6	

Fourth Week

	9 x 4 9 x 4 8 x 2 7 x 2 8 x 2
4 x 9	
	9 x 6, 9 x 7 7 x 6, 7 x 7 8 x 9, 8 x 6

Fifth Week

Clear

Final Test

Clear

It may seem surprising that the mistakes were not of greater frequency, but the length of time given to oral drill accounts for this. This experiment takes no account of the oral errors except to say that they were many, especially in the early part of the term. Leo had an aversion to examples and combinations which involved 6, 7, 8, and 9. This may have arisen from failure in previous performances. The nearness of the products of 7×8 and 6×9 ; 7×9 and 8×8 may have increased his mental confusion. It is worthy of note that about 100 of 165 errors involved these figures.

There was a very close connection between errors 8 and 9. Error 8 covered errors in the multiplication combinations, while error 9 included all intentional omissions in the multiplication or division combinations. The combinations 6×9 , 4×9 and 9×8 were missed most frequently. 6×9 and 9×6 were omitted intentionally most frequently. The division combinations related to difficult multiplication combinations and were often omitted. It is worthy of note that when the troublesome multiplication combinations were mastered difficulty with the reversals and with the related division combinations disappeared entirely. It would seem fair to assume that a transfer of learning had taken place.

During the third week Leo was given examples and combinations in division. The teacher felt that this change of attack

would provide variety and that a transfer of power from one process to another might occur. The plan was apparently successful as the fourth week showed an appreciable gain in accuracy. In the last week there was an outstanding gain in output and accuracy. Within a range of 237 possible errors Leo committed two.

In the Compass Diagnostic Test in Division the following results were obtained:

Part 1. His Division vocabulary rated a grading of high fourth.

Part 2. Division fundamentals were too low to score. Failure in the multiplication combinations produced failure in the division combinations. These failures were in the fourth quadrant of multiplication combinations.

Part 3. In short division with carrying Leo made a high fourth grading. However, the carrying habit was not fully established. There was an intentional omission of problems with divisors of 7, 8, and 9.

Part 4. This consisted of the addition, subtraction, and multiplication necessary to work the division which followed. Leo's grade was too low to score. There were some errors in zero combinations. Again consistent omission of multiplication com-

binations in which 6, 7, 8, or 9 were members.

Parts 5, 6, and 7. These parts consisted of long division. No attempt was made on these. This was anticipated as Leo had not had any fourth grade work.

During the five weeks only three mistakes were made in the addition combinations. These occurred during the third and fourth weeks. They were with the following combinations: $8+5$, $8+6$, $8+7$.

There were no errors in subtraction combinations during the entire term. They are as follows:

$18 \div 2$	$81 \div 9$	$11 \div 2$	$180 \div 9$
$12 \div 2$	$13 \div 2$	$14 \div 7$	$8 \div 2$

The chart on page nine shows the multiplication errors in quadrants of difficulty.

Daily descriptions of work accomplished follow. Comments on Leo's accuracy, growth in skills and industry, are included.

June 27

Work was almost entirely oral. Leo dawdled; minded other's business, delayed answer by several repetitions of teacher's questions.

June 28

Tables of 6's. Simple addition with no carrying, such as $10+4$, $10+5$, $10+2$.

June 29

Tables of 7's. Multiplication, (multiplier 6), involved carrying.

Short division, (divisors 5, 4, 8, 2, 3), involved carrying.

There was a decided increase in the quantity of work on this day. Leo was encouraged to check errors.

June 30

Oral drill on 7×7 , 7×8 , 7×9 .

Study of table of 7's.

Leo had a well developed habit of writing the carried number. The teacher let him do this as it seemed to help him. At that time there were many more important concerns to call to his attention.

July 1

On the last day of the week Leo's need for an even greater amount of oral work was evident. To check his habit of repeating the teacher's question several times he was made to say nothing but the answer. The boy's greatest weakness seemed to lie in an ignorance of the multiplication combinations. A new error also appeared; that of ignoring a number in the dividend if it did not contain the divisor.

To promote concentration the teacher planned to give Leo one (at least) problem daily, increasing as his work habits

improved. During this week the number of mistakes was surprisingly small but this may have been because so much of the work was oral and so much checking was secured. This week's successful accomplishment is as follows:

8 addition examples

11 subtraction examples

60 multiplication combinations

13 multiplication examples

11 division examples

In this work seven errors were noted; five multiplication combinations, and two division combinations.

July 5

Eighteen examples involving carrying.

Ten multiplication examples using as multipliers the digits 2-9.

Eighteen subtraction examples involving carrying.

Twelve addition examples, no carrying.

Two reasoning problems.

Multiplication work preceded by study of following combinations:

$$\begin{array}{ccc} 8 \times 8 & 6 \times 8 & 9 \times 7 \\ & 7 \times 9 & 7 \times 8 \end{array}$$

Leo's habit of reversing minuend and subtrahend was successfully avoided five times. His quantity of work greatly improved as his power of application grew.

July 6

Leo's mother asked to have his time divided between reading and arithmetic. It was not possible to do this, so his problem work was increased in the hope that his power in paragraph comprehension and word selection would develop further. These problems were supervised and considered successful if the process selection was correct. Any computation errors occurring in them were not considered in this paper as the numbers involved were extremely simple and as Leo's rate of accuracy was relatively high.

A considerable portion of this period was devoted to multiplication combination study, oral recitation and drill.

Eleven subtraction examples involving borrowing.

Eight multiplication examples with a four-digit multiplicand, and a one-digit multiplier (2-9).

July 7

Twelve subtraction examples

Nine multiplication examples

Two problems

Study and recitation of 4×9 , 4×8 , 6×9 .

July 8

The careless writing and lack of organization in Leo's papers became a new concern. Leo was urged to write larger and

to plan his paper. Evidences of effort in these matters appeared immediately, although the results left much to hope for.

Thirty-two examples in subtraction of the type shown below were worked correctly.

$$\begin{array}{r} 55 \\ -48 \\ \hline \end{array} \quad \begin{array}{r} 22 \\ -16 \\ \hline \end{array} \quad \begin{array}{r} 59 \\ -54 \\ \hline \end{array} \quad \begin{array}{r} 61 \\ -54 \\ \hline \end{array}$$

Ten multiplication examples with multiplicands of four digits, and multipliers of one digit (2-9) were completed without an error.

July 11

After a week's vacation from division Leo was confronted with it the third week. The divisor in each case was 2, but Leo was very unsuccessful. Ten of fifteen were incorrect.

To improve the size and legibility of his writing a portion of the period was devoted to copying numbers. To lengthen his time for computation he was given a third-grade work book and encouraged to proceed at his best pace. The beginning material in this book was comparatively simple and most of Leo's effort could be placed on the reading for comprehension. Five full papers were completed, and with one exception a high rate of accuracy was obtained between those attempted and those correct.

Attempted	Correct
26	25
44	43

Attempted	Correct
44	43
41	38
32	23

The low rate of accuracy on the last paper of the above list was due to an error in reading comprehension. After reading the direction aloud he successfully completed the nine which were wrong at first.

Samples of the type of work are:

Write the number that is 3 less than 21.

Write the number that is 2 more than 21.

Write the number of 5's in 10.

Additions such as 7 3 9 5

4 7 5 8

Leo showed gratification in producing a quantity of well-done material. It was the first morning that he apparently worked to capacity.

July 12

On this day Leo made 100 per cent on fourteen short divisions with four as a divisor. In his endeavor for accuracy he forgot all about form and the figures were almost illegible.

He completed three work sheets with a high degree of accuracy.

Attempted	Correct
41	41
34	34
33	31

The additions were very simple, two and three digit sums; adding with zero.

Samples of the work:

$$\begin{array}{r}
 7 \quad 2 \\
 0 \quad 5 \quad 5 \quad - \quad - \\
 0 \quad 1 \quad 6 \quad \frac{5}{9} \quad \frac{6}{9}
 \end{array}$$

Oral drill on the harder addition and multiplication combinations was carried on daily.

July 13

Leo had a perfect score paper which included thirty short divisions with divisors ranging from 2 to 9; twenty-four short division applications; eight division combinations. The desire to check work showed growth. The appearance of the papers showed improvement. The three work sheets were highly accurate.

46-44

54-54

46-41

July 14

Three work sheets with perfect scores were completed.

This work involved simple subtractions, time telling number selections. In the problem solving eleven were correct in thirteen attempted. There were two intentional omissions due to lack of reading ability. One error in computation appeared:

$$\begin{array}{r} 16 \\ \times 3 \\ \hline 28 \end{array}$$

July 15

A perfect score on the problem work was obtained. Seven correct in seven attempted. Four of five short divisions were correct. One was incomplete because of lack of time. In sixteen division combinations with a divisor of seven there was one error: $14 \div 7$

July 18

Leo worked correctly eight of ten problems; one was omitted; the error was that of selection of the wrong process.

Seven divisions of eight were correct. One, $(2471 \div 6)$, was intentionally omitted because of fear of the division combinations.

July 19

In eight problems three were incorrect. Two errors occurred in the selection of the wrong process, and one in an addition combination:

$$\begin{array}{r} 8 \\ +45 \\ \hline 54 \end{array}$$

Five short divisions of seven were correct.

Seven of seven multiplication examples, (with multipliers 6, 7, 8, and 9), were correct.

Leo showed growing ability to handle the multiplication combinations of the fourth quadrant.

July 20

Five of seven problems were correct. Two errors were in computations; forgot to carry; confusion of processes.

July 21

For several days papers had shown a development in tidy arrangement and legibility of writing. This day they were very untidy. His rate of accuracy was considerably lower than it had been.

Problems attempted

Problems correct

9

5

All reasoning errors in the above except one in division.

Problems attempted

Problems correct

5

4

The multipliers in the above were 6, 7, 8, and 9.

Problems attempted

Problems correct

10

7

The above were division problems.

Leo played ball too long and too hard before school and during recess. It was a very warm day. This may have occa-

sioned his decrease in accuracy.

July 22

Out of ten problems attempted five were correct. Two problems were not attempted, two reasoning errors, and one computation error.

In division nine of nine attempted were correct. The divisors ranged from 2-9.

In multiplication four of five were correct. The incorrect one is listed:

$$\begin{array}{r} 269 \\ \times 8 \\ \hline 2092 \end{array}$$

July 25

Leo was given examples in each of the fundamental processes.

The additions were three-column examples. Both were correct.

The two subtractions were four-column examples. Both were right.

In six division examples four were right. Errors occurred in division combinations.

In multiplication examples with two digit multipliers Leo got both wrong. He forgot to move the second partial product to the left. This was a new step for him. It was unfortunate that this had to be left until the last week of the term. One

problem correct in all five. All but one were reasoning errors.

July 26

Leo was one-half an hour tardy. Little written work was accomplished. He was given a study period on troublesome combinations, individual recitation time, and oral recitation time with two other students.

July 27

With dividends of five digits and divisors of 2 to 9, Leo worked correctly eight of eight. The work involved carrying. One problem correct in three. One error in these was in computation. It involved the moving of partial product to the left. This habit was not established.

In two and three column additions two correct of two attempted.

In four digit subtractions two correct of two attempted.

In two examples with two digit multipliers both were incorrect.

	July 28	
	Attempted	Correct
Addition	2	2
Subtraction	2	2
Multiplication	2	2
Very simple add.	27	27
Very simple sub.	26	21

	Attempted	Correct
Very simple mult.	26	25
Very simple div.	17	16

Leo advanced one month in paragraph meaning but slipped back three months in word selection. This was a slight disappointment, as it had been hoped that the power in problem-reading would transfer more emphatically.

The growth in the fundamentals was highly gratifying. In three of the processes a one and one-half year gain was registered. In division a one year gain was made. The teacher feels that Leo will make more than satisfactory progress if his drill work is properly organized and supervised, his study habits are guided, and much reading is offered to him.

From the observation of Leo at work and a detailed study of his papers the writer presents the following conclusions:

1. An I. Q. of 111 unaccompanied by sustained power of attention produced work which was low or mediocre in quality.
2. Incorrect answers to multiplication combinations were always the correct answers to some other combinations. This might indicate that interference to the learning of certain combinations is developed through the overlearning of others.
3. The mastery of certain multiplication combinations was accompanied by the mastery of the reversals of those combinations.
4. Daily work in problem reading developed power which

carried over into paragraph comprehension material of the final test.

5. The student's inferiority complex concerning multiplication combinations involving 7, 8, and 9 carried over into the division combinations involving the same digits.

6. The multiplication combinations were not equal in difficulty.

Arithmetic Test Scores, Educational Age, and Grade

Equivalent of Leo in Initial and Final Tests

Processes	Compass Survey		Educational Age		Grade Equivalent	
	Initial	Final	Initial	Final	Initial	Final
Addition	26	27	8-6	9-0	H 3	L 4
Subtraction	14	20	8-6	9-6	L 3	H 4
Multiplication	15	28	8-6	10-6	L 3	H 4
Division	3	12	8-0	9-6	L 3	L 4
Total	58	85	8-6	9-6	3.7	4.6

Reading Test Scores, Reading Age, and Grade Equivalent of Leo in Initial and Final Tests

	New Stanford Reading Test		Reading Age		School Grade	
	Initial	Final	Initial	Final	Initial	Final
Paragraph Meaning	20	23	7-5	7-10	2.6	2.7
Word Meaning	31	23	8-6	7-10	3.0	2.7
Total (average) Reading	25	23	8-0	7-10	2.8	2.7

Leo is a type that is met often in classroom teaching. He possessed good mental and physical powers, and his retardation at the beginning of the term was due to ineffective study habits. It is the opinion of the writer that overcrowded classrooms, with the resulting lack of individual attention, had produced this condition in Leo.

A study of Leo's test scores shows that he was not woefully retarded in any phase of arithmetic. Much stress on some of the multiplication combinations had so indelibly impressed their answers on his mind that he would give their products for those of other combinations. Flash responses had not been demanded of him and his repetition of the teacher's questions was done merely to gain time as he thought out the answers.

The teacher should be careful in her presentation of drill materials. Leo's difficulty with certain combinations and his facility with others may have resulted from some teacher's lack of recognition of the varying difficulty of the number combination and from lack of proper emphasis in their representation.

Keeping of a daily score of his work would help a student of this type to try to surpass previous records. Lengthening his addition columns would also increase his attention span. Permitting him to work with classes or grades of two levels

would enable him to progress and also "brush up" on poorly learned material. It would keep him occupied all the time and prevent his having unsupervised time in which to dawdle or idle.

Leo's type can be easily improved if the teacher keeps him busy with the proper drill materials. When his specific weaknesses are corrected and his study habits built up he should find himself in the upper third of his class.

Case Study of M. K.

Mary had finished second grade when she entered summer school. She was eight years old, and had an I. Q. of 111. Her excellent physical condition rendered a physical examination unnecessary. Mary had a very short attention span. Her memory span was decidedly limited. She would begin to work and in a few minutes the teacher would find her dreaming. She would sit for several minutes looking into space. After she became acquainted with the other students she would waste time whispering and giggling with them.

Mary's summer school attendance was irregular. Her attendance during the previous school year had also been irregular. The loss of content during her absences and her inability to understand the activities of her classmates may have encouraged Mary to sit doing little or nothing. She was a healthy little girl, an only child, who had everything she desired. She was sweet-tempered and apparently unspoiled.

The summer term, according to a comparison of the initial and final tests in arithmetic, gave her a two-month grade gain. This was evidenced by an increase in the total number of examples attempted and correctly finished. There was no gain which could be graded in addition and she remained at the high-second level in subtraction.

A seven-month grade gain was made in reading. According to the test used this advance was interpreted as a one and one-half year gain in reading age. This was due to a natural liking and ability for reading plus power gained in the work reading of arithmetic problems. In the reading Mary's short attention and memory span gave little trouble. The book was before her and the question could be answered immediately. In the forgetting of her addition and subtraction combinations the attention and memory weaknesses were obvious.

In the Compass Survey Test the student was ungradable in addition. Errors were made in the addition combinations, three-digit addition, and in the carrying concerned with higher decade addition. A score of eight toward a possible thirty was attained. Eight of twenty-three were correct. Many absurd answers were inserted to fill up space. The addition combinations errors occurred in the fourth quadrant of difficulty.

In the subtraction work a score of eight toward a possible twenty-five was made. The errors occurred in the subtraction combinations, confusion of processes, and absurdities. Nine of twenty-one attempts were correct. In some problems there was a combination of errors. The confusion of processes and absurdities were caused by lack of automatism with subtraction and combinations and inability to borrow in subtracting. The latter limitation was not surprising as many children have no

training in this until the third grade.

The other parts of the test were graded zero, and as they were beyond Mary's grade experience they are not discussed here.

In the Compass Diagnostic Test in the addition of whole numbers Mary was ungradable in the basic additions facts, higher decade addition, column addition, carrying in column addition, and checking answers in addition. The retardation in the learning of the basic addition facts was a matter of concern. The other limitations were not so vital as many schools place comparatively little emphasis on more than the addition and subtraction combinations during the first two years.

Toward a possible score of 70 the student made a score of 39. Sixteen combinations were omitted and fifteen were incorrectly answered. The difficulty lay in all quadrants, but the greatest number of inaccuracies belonged in the fourth quadrant.

In higher decade addition lack of skill with the addition combinations was displayed; no knowledge of carrying was apparent; absurdities, committed because of the child's desire to write something, were noted.

June 27

Mary was given eleven examples in very simple addition involving no carrying. She made a grade of 100. Samples of

the work follow:	25	46	23
	43	51	44
	<u>68</u>	<u>97</u>	<u>65</u>

She was given problems involving money writing. The results were highly accurate. Problem work was given consistently during the term. One of the reasons for the assignment of problem work was to encourage a possible transfer of power in problem reading to other work type reading.

June 28

Mary was given ten examples in column addition. They involved no carrying. Three of ten attempts were correct. Mary counted to get answers. Need for drill in combinations, even those under ten, was imperative.

Ten simple problems were answered correctly. Mary used very good judgment in answer selection after the reading of the problem.

June 29

Mary was given part of the period to practice the making of dollar signs. The period was two hours long with a twenty-minute recess after the first hour. Mary's work had to be varied several times during the morning as she became fatigued and bored quickly. The teacher felt that the work of the preceding day had been too difficult. Mary was given eight combinations to answer. These were handled accurately. Counting may have occurred, as the teacher was working with another student while Mary was occupied.

Two-column additions similar to those of the preceding day were then given. One was worked correctly.

Mary was given an opportunity to study a few combinations with the answer before her. She then recited these. Before giving a detailed daily report of the pupil's work a list of the error types observed in an analysis of her papers is presented.

1. Mistake in addition combinations

$$\begin{array}{r} 7 \\ 6 \\ 11 \end{array}$$

2. Error in "seen" to "thought of" number.

$$\begin{array}{r} 8 \\ 3 \\ 6 \\ 15 \\ \hline 22 \end{array}$$

3. Carried unnecessarily

$$\begin{array}{r} 5 \\ 13 \\ \hline 28 \end{array}$$

4. No knowledge of the carrying process.

$$\begin{array}{r} 8 \\ 13 \\ 111 \end{array} \quad \begin{array}{r} 9 \\ 29 \\ 218 \end{array} \quad \begin{array}{r} 16 \\ 6 \\ 112 \end{array} \quad \begin{array}{r} 11 \\ 9 \\ 110 \end{array}$$

5. Absurdities that could not be classified.

$$\begin{array}{r} 11 \\ -5 \\ \hline 3 \end{array} \quad \begin{array}{r} 18 \\ 6 \\ 012 \end{array} \quad \begin{array}{r} 4 \\ 16 \\ 011 \end{array} \quad \begin{array}{r} 7 \\ 13 \\ 11 \end{array}$$

6. Errors in subtraction combinations.

$$\begin{array}{r} 13 \\ 9 \\ \hline 10 \end{array} \quad \begin{array}{r} 37 \\ 24 \\ \hline 12 \end{array}$$

7. Confusion of processes.

$$\begin{array}{r} 107 \\ 98 \\ \hline 1915 \end{array}$$

8. Reversal of minuend and subtrahend.

$$\begin{array}{r} 400 \\ 181 \\ \hline 380 \end{array}$$

9. Ignorance of borrowing process in subtracting.

$$\begin{array}{r} 623 \\ 225 \\ \hline 403 \end{array}$$

The addition combinations which gave difficulty in the diagnostic tests are arranged in quadrants:

3+4	2+5	6+4	9+4	7+6	7+5
2+4	4+5	7+4	7+3	9+5	7+5
	3+7			5+8	8+6

She was allowed to hear a fellow student recite. These two worked together a good part of the term. He helped Mary with the combinations. She read the problem for him many times.

June 30

In an exercise in money writing three of five were correct. Mary was given help in "cent" and "dime" writing.

In twelve addition examples of two-columns involving no carrying two were correct. The errors occurred in adding "seen" to the "thought of" number. This error type was vitally related to the mistakes in addition combinations.

Ten addition examples with carrying but involving the

and Percentage of Errors

Error	1	2	3	4	5	6	7	8	9
Preliminary Diagnostic	6 15 40 15 54 21.7 6 64 9.3	6 17 35.2	1 1 100	4 4 100 5 12 41.6	3 - -	3 39 7.6	1 - -	4 - -	1 7 142
28		8 70 11.1							
29		1 7 14.2							
30	4 20 20	8 84 9.5							
1	5 56 8.9								
5		4 18 22.2							
8	5 42 11.8	1 11 9.0							
6,11,12,13,14	No written errors								
15	1 10 10					5 40 12.5			
18	4 40 10			2 4 50					
19	2 20 10								
	13 40 32.5								
7, 20	Absent								
21	1 20 5	2 40 5	3 23 13	3 52 5.7					
22	8 20 40 1 4 25 3 40 7.5								
25	10 40 25 18 40 4.5					3 40 7.5 1 40 2.5 8 40 20 1 40 2.5 4 40 10			
26	3 20 15 5 12 41.6	2 12 16.6 3 18 16.6 3 142 1.4							
27	3 42 7.1 2 40 5 2 8 25					3 12 25 5 20 25			

TABLE XI (continued)

		Addition Combinations			Addition No Carrying			Addition With Carrying			Subtraction Combinations			Problems		
July	22	20	12	60							40	36	90	4	3	75
		40	37	92.5							40	35	87.5			
	25	16	16	100							16	16	100	11	11	100
		40	40	75							40	37	92.5			
		40	22	55							40	32	80			
											40	35	82.5			
	26	20	17	85				14	10	71.4	40	38	95	8	8	100
		18	15	83.3	2	2	100									
	27										80	54	67.5	8	7	87.5
											40	38	95			
Final		12	10	83.3	8	1	12.5	2	0	0	8	5	62.5			

handling of only two digits per column showed a higher rate of accuracy than the type just described. Six of ten attempts were correct. The carrying process was perfectly handled even in those that had incorrect answers. The inaccuracies occurred in the addition combinations. The combinations were those of the fourth quadrant.

July 1

Mary was given an exercise in recognizing number groups. A little ability in work reading was necessary for the successful completion of this work. Three of three attempts were handled perfectly.

The week's drill on addition combinations under ten showed its value, though slightly, on this day. Three of eight examples in column addition were correct. Although the grade was not much higher, an additional improvement was noted. In most instances the error occurred in but one column. Heretofore both columns were added incorrectly in many cases.

July 5

Twelve of twelve addition examples involving no carrying merited 100. The examples had two digits in a column. The demand on computational power was merely the knowledge of the combinations under ten.

One of five addition examples was correct. The difficulty lay in the adding of a "seen" to a "thought of" number. The

carrying process was fairly well handled.

Twenty-five of twenty-five subtraction combinations with minuends under ten were perfectly answered.

July 6

Six of six two-column addition examples with no carrying and with three digits in each column were completed without error.

July 7

Mary was absent on this day.

July 8

Fifteen of twenty-one additions involving carrying and with two digits to a column were correct. Five errors were due to inaccuracies in the combinations. One mistake occurred in the confusion of processes.

Four of five longer column additions involving carrying were correct. The error occurred in adding a "seen" to a "thought of " number.

July 11

Two hundred thirty of two hundred forty addition combinations, ten and under, were correct. These combinations were taken from Washburne's "Individual Book One." The two hundred forty combinations worked on this day covered the first six steps of difficulty as arranged by Washburne.

July 12

One hundred twenty of one hundred twenty addition combinations covering Washburne's next three steps of difficulty were 100.

July 13

In each of four practice tests which included the combinations studied on the preceding days a 100 per cent grade was made.

In a set of five problems involving addition and subtraction two were correct. The teacher felt that the assignment had been too difficult.

July 14

Four of four problems involving addition facts under ten were 100.

Ten of eleven addition combinations were correct. Nine of these had sums over ten.

Thirty-nine of forty addition combinations in Washburne's Step Ten were correct.

Thirty-nine of forty addition combinations in Washburne's Step Eleven were correct.

Thirty-nine of forty subtraction combinations of Step one difficulty were perfect. The one error occurred in the confusion of processes.

July 15

Eight of ten problems involving the addition combinations with sums of ten and under were correct. One error occurred through ignorance of $7+3$. The other occurred in the selection of the process.

Thirty-six of forty combinations in Step Twelve of difficulty were correct.

Daily oral drill was given to Mary in the combinations.

Thirty-five or forty subtraction combinations of Step Two were correct. Four errors involved the same combination. The other error occurred in $7+3$. Mary previous had had trouble with $7+3$.

Thirty-five of forty addition combinations of Step thirteen were correct. The errors occurred in the fourth quadrant of difficulty.

Thirty-eight of forty addition combinations of Step Fourteen were correct.

In two tests 100 of 100 addition combinations were correct.

July 19

Eighteen of twenty addition combinations with sums over ten were correct.

Mary had a chart of combinations arranged according to difficulty. These were studied each morning, recited, restudied, and written.

Six of eight addition examples involving carrying were wrong. The two errors occurred in the carrying process.

Twenty-seven of forty addition combinations of Step 16 were correct.

Four of four problems were correct.

July 21

Nineteen of twenty addition combinations were correct.

The teacher gave Mary much oral help and drill. One of the older students listened to her addition and subtraction combinations after the teacher worked with her.

Twenty-three exercises using different forms of addition were given to Mary. Twenty were correct.

Four of four problems were worked correctly.

Forty-nine of fifty-two addition combinations in a test were correct.

Thirty-eight of forty addition combinations in a test were correct.

Thirty-three of forty addition combinations in a test were correct.

July 22

Twelve of twenty addition combinations were correct. The errors occurred in the fourth quadrant of difficulty.

Thirty-six of forty subtraction combinations were correct. These combinations were in Washburne's Step Three of difficulty.

Three errors concerned the same combination: $7\frac{1}{4}$.

Four of three problems were worked correctly. The error occurred in the addition combinations.

Thirty-seven of forty addition combinations were correct.

Thirty-five of forty subtraction combinations in Step Three were correct.

July 25

Eleven of eleven problems in addition combinations were 100. The "and What" form of addition was presented to the student as a preparation for subtraction.

Sixteen of sixteen "and what" addition combinations were correct.

Thirty of forty "and what" addition combinations were correct.

Twenty-two of forty "and what" addition combinations were correct.

Sixteen of sixteen subtraction combinations were 100.

Thirty-seven of forty subtraction combinations in Step Four were correct.

Thirty-nine of forty subtraction combinations in Step Five were correct.

Thirty-two of forty subtraction combinations in Step Six were correct.

Thirty of forty subtraction combinations in Step Seven

were correct.

Four errors were due to confusion of process with addition.

July 26

Nineteen of twenty examples in addition were correctly finished.

Five of twelve bills involving addition were added correctly.

There were five combination errors and two errors in adding a "seen" to a "thought of" number.

Fifteen of eighteen single column additions were correctly computed.

Ten of fourteen examples involving carrying were correctly solved.

Two of two column additions without carrying were correct.

Thirty-nine of forty-two addition combinations were correctly solved.

Thirty-eight of forty subtraction facts of Step eight in difficulty were correct.

Eight of eight problems in subtraction facts were correctly solved.

July 27

Seven of eight problems in subtraction facts were correctly worked. The error occurred in an addition combination of a problem which had three parts.

Twenty-seven of forty subtraction facts of Step Ten were accurately answered.

Mary was given study periods every day. During the first part she studied combinations with answers before her. Then she covered the answers and tried to remember them. If she could not she looked. The teacher and one of the students listened to her several times each morning.

Twenty-seven of forty subtraction combinations of Step Eleven were answered accurately. Although thirteen examples were missed these represented eight combinations, as some combinations were repeated several times.

July 28

Thirty-eight of forty subtraction combinations of Step Twelve were correctly answered.

In an addition and subtraction review ten of twenty examples were correct. Confusion of processes occurred four times. There were two subtraction combinations, and two addition combination errors.

August 1

In the final test the following results were obtained:

In addition fourteen was made toward a possible score of thirty. This was six more than had been reached in Form A of the same test at the beginning of the term. Five of the errors were combination errors and six were "seen" to "thought of"

number inaccuracies. This score did not permit a grade interpretation so the student ended the term still ungradable in addition.

In addition a score of nine was made toward a possible twenty-five. This was an improvement of one over the first test. It did not raise the grade interpretation of her first mark. Four errors occurred in the combinations. Five errors were absurdities that could not be classified. Five errors occurred in the reversal of minuend and subtrahend through the child's ignorance of the borrowing process.

Arithmetic Test Scores, Educational Age, and Grade

Equivalent of Mary in Initial and Final Tests

Processes	Compass Survey		Educational Age		Grade Equivalent	
	Initial	Final	Initial	Final	Initial	Final
Addition	8	14	-	-	-	-
Subtraction	8	9	7-6	-	H 2	H 2
Multiplication	0	0	-	-	-	-
Division	0	0	7-0	7-0	-	-
Total	16	23	-	-	2.4	2.6

Reading Test Scores, Reading Age, and School Grade

of Mary in Initial and Final Tests

	New Stanford Reading Test		Reading Age		School Grade	
	Initial	Final	Initial	Final	Initial	Final
Paragraph Meaning	19	34	-	8-9	-	3.1
Word Meaning	21	43	7-6	9-5	2.6	3.6
Total (average) Reading	20	38.5	7-5	9-1	2.6	3.3

After observing Mary at work and judging from an analysis of her papers these conclusions seem justified:

1. An I. Q. of 111 unaccompanied by sustained interest results in meager achievement.
2. Inadequate initial learning of addition and subtraction combinations was partly responsible for Mary's failure.
3. A transfer in reading power from problems reading to work reading of another type was evident.
4. Mary had relied on counting for some time. This had been successful for sums under ten. She was lost when the sums were over ten.
5. Counting "back" in subtraction was a procedure Mary indulged in. Her apparent success in subtraction may have been attained because of the decreasing size of the remainder.
6. A lack of "arithmetic Readiness" may have hindered her progress.

As Mary was just eight years old and had finished second grade the writer feels it was unwise to start (during the summer) work of the remedial type. The parents would have been wiser if the child had been permitted a vacation of pure recreation. So many schools are not stressing arithmetic until the third grade that a little longer time might have been given to Mary to "find" herself.

Case Study of P. M.

Paul had finished second grade when he came to summer school. He was just eight years old. He had an I. Q. of 115. He seemed to be a strong, sturdy little boy, and for this reason the physical examination was omitted. During the term he contracted a poison oak infection which held him back in application for a few days. He was registered in the arithmetic group, but the teacher felt that his needs in reading were more urgent than in arithmetic. Paul was lost in problem work because of his lack in reading ability. When the problems were read to him he showed good judgment in the selection of the process and accuracy in the computation involved. He had little power in word discrimination.

His growth in arithmetical power during the summer term was highly gratifying. At the close of the term Paul's parents were informed that he could still profit by drill in the addition combinations, especially those that had sums over ten. It was also suggested that he begin working for speed.

In the Compass Survey Test in Arithmetic Paul made a total score of twenty-nine which was interpreted as placing him in grade 2.8. In addition the score placed him in high second grade. Eighteen of twenty-three attempts in addition were correct. There were two intentional omissions. Three errors

occurred because of ignorance of the carrying process. There were no errors in combinations.

A grade of high second was scored in subtraction. Nine of seventeen attempts were correct. Two errors were made in the subtraction combinations. Five errors occurred in the reversal on minuend and subtrahend. The ignorance of the borrowing process in subtraction was responsible for this procedure. There was one intentional omission.

The diagnostic test in addition of whole numbers was then administered. The total score on this test could not be interpreted in terms of grade placement. Six addition combinations were intentionally omitted. All attempts were accurately answered.

In higher decade addition twenty-nine of forty-seven attempts were correct. There were six intentional omissions. All other errors but one were due to ignorance of the carrying process. One error occurred in the combinations.

In column addition with no carrying three of three attempts were correct. In column addition with carrying there were ten attempts but none was correct. Ignorance of the carrying process reduced this score.

The diagnostic test in subtraction was also administered. No part of this score could be interpreted in terms of grade. There were sixteen intentional omissions in the basic subtraction facts. Twenty-two of twenty-four attempts were correctly

answered. The errors occurred in "zero" combinations.

In harder subtraction two of twenty-three attempts were correct. Most of the answers were incomplete or absurdities which could not be classified. No successes were recorded in checking answers or finding errors in subtraction as suggested by parts 3 and 4 of the test.

A daily survey of Paul's papers revealed eight types of errors. A description and sample of each follows:

1. Intentional omissions:

$$\begin{array}{r} 8 \\ 7 \\ \hline 19 \end{array}$$

2. Ignorance of carrying process:

$$\begin{array}{r} 3 \\ 29 \\ \hline 212 \end{array} \quad \begin{array}{r} 6 \\ 38 \\ \hline 314 \end{array} \quad \begin{array}{r} 7 \\ 45 \\ \hline 412 \end{array}$$

3. Error in subtraction combinations:

$$\begin{array}{r} 11 \\ 4 \\ \hline 13 \end{array} \quad \begin{array}{r} 13 \\ 9 \\ \hline 16 \end{array}$$

4. Reversal of minuend and subtrahend because of ignorance of the borrowing process.

5. Error in addition combinations:

$$\begin{array}{r} 6 \\ 2 \\ \hline \end{array}$$

6. Error in adding a "seen" to a "thought of" number:

$$\begin{array}{r} 63 \\ 11 \\ 29 \\ \hline 95 \end{array}$$

TABLE XII
Number of Problems Attempted by Paul; Number Right;
and Percentage of Accuracy

[illegible]

TABLE XII (continued)

158

	Addition Combinations			Addition No Carrying			Addition With Carrying			Subtraction Combinations			Subtraction No Borrowing			Problems		
18	40	38	95													8	8	100
	24	24	100															
	40	40	100															
	40	37	92.5													4	4	100
19	40	39	97.5															
	40	40	100															
	10	8	80															
	10	10	100															
20	33	33	100							27	21	77.7				20	20	100
	16	15	93.7													3	2	66.6
	100	100	100															
21	39	39	100													18	18	100
	33	33	100															
22	40	38	95							40	39	97.5				9	8	88.8
										40	35	87.5						
25										40	37	92.5						
										40	40	100						
										40	36	90						
26				16	15	93.7	12	12	100	40	29	72.5				4	1	25
							4	4	100									
27	7	7	100	7	7	100	12	12	100									
	10	10	100	25	24	96												
				10	10	100	13	0	0									
				14	14	100	16	14	87.5	4	4	100						
Final	10	10	100							8	6	75	10	10	100			
													5	5	100			

TABLE XIII
Opportunities of Paul for Errors of 8 Types by Days;
Number of Errors Made; and Percentage of Error

Error	1	2	3	4	5	6	7	8
Preliminary	2	3 3 100	2 7 28.5	5 5 100				
Diagnostic	6	12 12 100			1 94 1.0			
	16		2 24 8.3					
27								
28		1 1 100			1 10 10			
29					2 12 16.6			
30								
1		2 5 40						
5		5 5 100	5 30 15.6					
6			2 16 12.5			1 22 4.5		
7		1 6 16.6				1 63 1.5		1 18 5.5
8								
11					1 200 .5		16 240 2.5	
12		2 40 5						
		8 40 20						
		2 40 5						
13								
14								
15					2 40 5			
					6 40 15			
18					2 40 5			
19					1 40 2.5			
					2 10 20			
20								
21								
22					1 40 2.5			
					5 40 12.5			
25					3 40 7.5			
					4 40 10			
26					1 4 2.5			
27		13 0 100						

7. Confusion of processes.
8. Carrying the wrong number:

29
11
52

A description of each day's work is now presented.

June 27

Thirty-four of thirty-four addition combinations under 10 were correctly answered. The speed of response needed stimulation.

June 28

In the ten addition examples of four two-digit numbers each eight were correct. The first error occurred in the carrying process. The second was due to a failure in the addition combinations.

Five of nine problems were correctly solved. The errors were due to a wrong selection of process. This selection was the result of Paul's weakness in reading. When the problem was read to him he gave the correct answer immediately.

June 29

The carrying process was explained to Paul. He was then given a study period on the combinations above ten. This was followed by assignment of thirteen additions of two two-digit numbers. These examples involved carrying. Thirteen of Thir-

teen were accurately worked.

Paul wrote a short assignment which included long and short ways of writing money. His work was neatly written, well arranged, and highly accurate. It was a pleasure to mark his papers. These characteristics were well developed on his entrance to summer school.

June 30

Ten of twelve additions of two two-digit numbers each had correct sums. Both mistakes were errors in the combinations. The carrying process caused no trouble.

Four of four problems were graded 100.

Ten of eleven efforts in the writing of money were correct.

July 1

Most of this period was given to study of the combinations above ten, oral recitations to the teacher, and to a student.

Three of five problems were right. Two errors were due to failure in carrying. Paul needed very simple problems. The reading difficulty in problems was greater than the mathematical difficulty.

July 5

Twelve of twelve additions of two two-digit numbers which involved no carrying were accurately computed.

Two of two problems were correctly solved.

Five additions involving carrying were wrong. All notion of the carrying habit had vanished over the three day weekend. The exercise had no errors in combinations.

Thirty subtraction combinations were then given. All attempted were accurately answered. Five were intentionally omitted. These are listed throughout this study as errors.

July 6

Fourteen addition examples of four two-digit numbers each were graded 100. These examples involved no carrying.

Five of six addition examples with carrying were correct. The error occurred in adding a "seen" to a "thought of" number.

An assignment of sixteen subtraction combinations was then given. Fourteen responses were perfect.

July 7

On this day Paul worked eighteen subtraction examples. Minuend and subtrahend of each example ended in zero. Seventeen were computed correctly. The error occurred in the confusion of processes.

The one problem assigned was correctly worked.

Eighteen addition examples involving borrowing were in the last assignment of the day. One error occurred in the adding of a "seen" to a "thought of" number. The second error was that of carrying the wrong number.

July 8

Twenty additions of two two-digit numbers were perfectly completed. Carrying was used.

Six longer additions involving carrying were graded 100.

Six of seven subtraction examples were correct. The error made occurred in the confusion of the processes. Both minuend and subtrahend ended in zero.

July 11

At the beginning of the third week Paul began work in Washburne's Individual Arithmetic: Book One. The addition and subtraction facts are each divided into sixteen steps of difficulty in this book.

Forty of forty addition facts of Step One were perfectly answered.

Thirty-six of forty addition facts of Step Two were correctly answered. The same error occurred forty-four times. The answer to one plus one was given as zero. A confusion of processes was responsible for this error.

Thirty-nine of forty additions of Step Three were correct. The error which occurred in Step Two appeared in Step Three.

In the forty addition facts of Step Four a grade of 100 was made.

In the forty addition facts of Step Five one error appeared. Again it was one plus one equals zero.

Sixteen subtraction facts received perfect responses.

Thirty-nine of forty addition facts of Step Six were correct. The one error was a combination error.

July 12

Paul was able to handle many more combinations when he used the workbook. All that he wrote was the answer to each combination. The one hundred sixty combinations of steps seven, eight, nine, and ten in addition facts received correct responses.

Work in the subtraction facts was attempted in steps one, two, and three. In Step one, two subtraction errors were noted. Thirty-eight of forty responses were correct.

In Step two there were eight inaccuracies. These eight errors included four combinations as two of the combinations were missed three times each.

July 13

Four of four problems from the Individual Arithmetic were correct.

Four tests of forty combinations each in facts under ten were perfect.

July 14

Paul was absent on this day. He had contracted poison oak. For a few days it was hard to hold his attention as the

itching of his fingers annoyed him so.

July 15

Four of five problems were solved correctly. The incorrect one had an absurd answer.

Thirty-eight of forty addition combinations of Step ten were correct. The same error occurred twice.

Thirty-four of forty addition combinations of Step 11 received correct responses. Paul repeated the same incorrect answers for certain combinations. The problem seemed to be the removal of some previous incorrect impression.

July 18

Thirty-eight of forty addition facts received correct responses.

In an exercise involving addition facts in all forms the student made a grade of 100.

In two practice tests of addition facts under and over ten the grades were respectively 100 and 93. Three combination errors were found.

Eight of eight problems involving addition facts were correct.

July 19

Thirty-nine of forty addition facts of Step thirteen were answered correctly.

Four of four problems in addition facts were solved accurately.

Forty addition facts of a test in the Individual Book were graded 100.

Oral work was being carried on consistently in the subtraction facts of the Steps in the Individual Book of Washburne's. Special emphasis was placed on those combinations missed in the different exercises.

Eight of ten addition combinations which had given trouble on previous days were correctly answered in a test.

Ten of ten combinations which had previously presented difficulty received correct responses.

July 20

Thirty-three of thirty-three addition combinations were answered correctly.

Twenty of twenty problems in addition facts were solved perfectly.

Twenty-one of twenty-seven subtraction facts received perfect responses. Four were intentionally omitted.

Two of three problems were solved correctly. Incorrect one was due to an error in addition combinations.

Fifteen of sixteen "and what" combinations were correct.

One hundred addition facts of Washburne'd Addition Table were graded 100.

Thirty-one of forty "and what" combinations were answered correctly. The process was not understood.

July 21

The combinations which had given trouble were grouped, studied, recited, and tested. Thirty-nine of thirty-nine answers were right. Some combinations were repeated in the test.

Eighteen of eighteen problems were solved correctly.

Thirty-three of thirty-three addition facts were answered correctly.

The "and what" problem was explained to the student. Thirty-eight of forty combinations were correct.

July 22

Eight of nine problems were correctly solved. The error in the incorrect problem could not be classified.

Thirty-nine of forty subtraction facts of Step Four were answered correctly.

Thirty-five of forty subtraction facts of Step Five were correct. In the five errors three combinations were represented.

July 25

Thirty-seven of forty subtraction facts of Step Six were accurately answered.

Forty of forty subtraction facts of Step seven were graded 100 per cent.

Thirty-six of forty subtraction facts of Step Eight were perfectly answered.

Four of four problems involving the subtraction facts were solved perfectly.

Oral drill on the subtraction combinations and the addition combinations which had given trouble was pursued consistently.

July 26

One of four problems involving subtraction facts was correct. Two errors occurred in the confusion of processes. One error was due to a failure in the subtraction combinations. Twenty-nine of forty subtraction combinations of Step Ten were correctly answered. Six of these errors were intentional omissions.

Fifteen of sixteen examples in single column additions were accurately computed.

Twelve of twelve additions of two two-digit numbers were correct. Carrying occurred in these examples.

Four of four additions of three two-digit numbers received accurate responses.

July 27

Twelve of twelve addition examples involving carrying were correctly added.

Seven examples in single addition were graded 100.

Seven of the addition combinations which had caused trouble received correct responses.

Twenty-five examples of graded difficulty in addition contained but one error. This occurred in an addition combination.

In a diagnostic Chart for Fundamental Processes in Arithmetic by Buswell and John the student attempted thirty-two examples. Nineteen of these attempts were correct. Two were incomplete. Eleven errors were due to the forgetting of the carrying process. Not once was it used. The appearance of the chart may have made the child "examination conscious." Several of the examples involved two borrowings. Even in the simple examples no use was made of the skill he had developed during the term.

Four of four problems concerning the subtraction facts were accurately solved.

Twenty-eight of forty subtraction facts of Step Ten received accurate responses. This step included combinations in which the subtrahend was zero. Ten of the errors were in the "zero combination" type.

July 28

On this day the Diagnostic Chart of the day before was shown to the student. He was given an informal talk concerning the carrying process. A new copy of the same chart was then

given to him. This time thirty-two of thirty-six attempts were correctly computed. Only twice did the student forget to carry. The other two examples involved several carryings beyond the student's power. One of these examples was the addition of two seven-digit numbers. The boy correctly added a thirteen-digit column. This was a splendid example of sustained attention.

Fourteen of fourteen attempts in subtraction were correct. The teacher regretted that she had not been able to present "borrowing" to the child. Three attempts in multiplication were correct. No training had been given in multiplication. The results were apparently a transfer of power from addition.

An exercise in all "forms" of addition and subtraction was given. Twenty-one of twenty-four answers were right. Confusion of the processes caused two errors. A subtraction fact error caused the third inaccuracy.

In Form B of the same test that was used at the beginning of the term the following results were obtained:

In addition a score of 27 toward a possible 30 was obtained.

In subtraction a score of 11 toward a possible 25 was obtained.

In multiplication a score of 8 toward a possible 35 was obtained.

In division a score of 1 toward a possible 25 was obtained.

In addition all but one was attempted. The two errors

occurred through ignorance of the borrowing process.

The ability to score anything in the multiplication and division was a pleasant surprise and gratification to the teacher. A gain of one year was shown in addition. The subtraction grade remained stationary. No gradation was possible at the beginning of the term in multiplication. The gradation of high second was a distinct achievement.

In the diagnostic tests in addition and subtraction at the beginning of the term the student was ungradable. A comparison of this record with the final records showed distinct progress on the part of the student. The teacher was left with the question of whether the borrowing process should have been introduced. One reason against it was the fact that only ten of the sixteen steps of difficulty in subtraction had been presented.

Arithmetic Test Scores, Educational Age, and Grade

Equivalent of Paul in Initial and Final Tests

Processes	Compass Survey		Educational Age		Grade Equivalent	
	Initial	Final	Initial	Final	Initial	Final
Addition	20	27	7-0	9-0	H 2	H 3
Subtraction	9	11	7-6	8-0	H 2	H 2
Multiplication	0	8	-	8-0	-	H 2
Division	0	1	7-0	7-6	-	-
Total	29	47	7-0	8-0	2.8	3.3

Reading Test Scores, Reading Age, and School
Grade of Paul in Initial and Final Tests

	New Stanford Reading Test		Reading Age		School Grade	
	Initial	Final	Initial	Final	Initial	Final
Paragraph meaning	16	19	-	-	-	-
Word meaning	16	19	-	-	-	-
Total (average) Reading	16	19	-	-	-	-

From the observation of Paul at work and from a study of his papers the writer presents the following conclusions:

1. A growth in the attention span showed itself in the ability to add increasingly longer columns.
2. Reading difficulties produced failure in problem work.
3. There was a transfer of power from the addition combinations to the multiplication combinations. The latter had not been taught but were successfully attempted in the final examination.
4. A written error made a definite impression on Paul. Many attempts were needed to eradicate an erroneous notion from his mind.
5. Many of his troubles would never have occurred if his previous teacher had normal classroom membership.

The delaying of remedial work in arithmetic would have been

a wiser procedure. A summer of natural growth might have done as much for the student. Help in reading was also essential but the student did not have time for two subjects.

CHAPTER IV

CONCLUSIONS

CHAPTER IV

1. Conclusions Concerning Types of Children in Need of Remedial Work

- a. George represented the group that will never rise above mediocre achievement even under the most favorable conditions. His I. Q. was one point below that considered normal. Under unfavorable conditions the achievement of this type of pupil falls far below normal.
- b. Shirley belonged to that class of gifted children whose achievement, because of interfering factors, falls far below the standard expected from the I. Q. She was timid, suppressed, and a day dreamer. Previous failures or perhaps a mental conflict not apparent to the teacher showed in a mere endurance of all class work. No enthusiasm for praise nor depression from adverse criticism was ever evidenced by the pupil.
- c. Margaret was a child of normal intelligence. She was extremely sensitive, easily depressed, overly motivated, and morbidly anxious about her progress. Her achievement, in view of her mental endowment, was quite remarkable. What she paid for this progress from the standpoint of health, nervous stability, and peace of

mind is unmeasured, but the teacher feels the price was excessive.

- d. Leo belongs to the group who possess good minds and good bodies. He had developed a set of ineffective study habits the results of which disastrously influenced all phases of his achievement.
- e. Paul and Mary were the victims of overly solicitous parents. Truly enough, they were below grade in the subject studied, but the writer feels that time with its accompanying natural growth could have produced a development almost equal, at least, to that attained in the remedial class. There is reason for believing that such a course may be unwise in that it makes the child feel that the subject is difficult and that he is decidedly inferior to his classmates in the subject.

2. Conclusions Concerning the Effects of Mental States on Achievement

- a. With children of meager mental endowment the best that can be expected is achievement of mediocre quality.
- b. With the gifted child who is battling some inner conflict the achievement is likely to be very disappointing in quality and in quantity. Very little improvement and occasionally regression in a subject or a phase of a subject is likely to occur. With children of high

- I. Q's a mental conflict is more apt to be present than those with low I. Q's.
- c. An inferiority complex established in a child of high I. Q. can defeat regular attendance, earnest teaching, and a sincere effort to change the attitude.
 - d. The highly interested child of ordinary mental endowment often produces results that, in the light of educational age and grade equivalents, are exceedingly gratifying.
 - e. Good mental and physical endowment, unaccompanied by helpful study habits and attitudes, produces results that are mediocre and often below normal.
 - f. The reaction of aversion to criticism, especially with a sensitive child, is often evidenced in the presentation of alibis and excuses and a desire to hide from defeat.

3. Conclusions Concerning Major Arithmetic Weaknesses of the Cases of This Study

The errors listed below were generally committed by the members of this group. The first three types listed adversely influenced the achievement of all and were considered by the teacher to be the most grave of those discovered. Unless these three error types could be removed little could be expected in the way of arithmetic accomplishment of even primary grade level

The other seven are listed in graded order as the teacher felt they lowered the quality of achievement most frequently.

1. Errors due to lack of automatism in some combinations of all processes.

2. Errors due to lack of automatism in certain combinations the inherent difficulty of which educational research acknowledges.

3. No desire, and in some cases, positive aversion to the checking of work.

4. Errors in higher decade addition.

5. Errors in adding a "seen" to a "thought of" number.

6. Omissions and commissions in the carrying process.

7. Omissions and commissions in the borrowing process.

8. Errors resulting from the confusion of processes.

9. Errors involving the manipulation of zeros.

10. Errors due to incorrect copying.

4. Data on the Improvement of the Subjects of These Studies

George stood still as far as grade equivalent in arithmetic was concerned. He regressed one year in addition and was ungradable in division at the end of the term although he had made a grade score of H-3 at the beginning. He stood still in multiplication. He made a score in subtraction which was interpreted as a grade equivalent of H-4. This was his only sign of progress in arithmetic as he had been ungradable in the be-

ginning. In reading, his paragraph meaning made a one-month grade gain but in word meaning he made no progress.

Shirley made slightly higher arithmetic scores at the end than at the beginning of the term in all processes except division. She was still ungradable, according to the norms of the test used, in every process except multiplication in which she attained proficiency which placed her at low-fifth level. Her total arithmetic score gave her a grade equivalent of 4.2 at entrance and 4.6 at leaving. She missed the initial reading test but in the final test made a score which placed her in grade 4.5

Margaret made a one-year gain in arithmetic. At the beginning of the term she was ungradable in addition and in problem work. At the end she reached a grade equivalent of high-eighth in addition, and low seventh in problem work. She retained the same grade level (H-5) in subtraction and multiplication but made a three-year gain in division advancing from H-5 to H-8. On entrance her reading grade was 3.9. Her final test gave her a grade equivalent of 5.85 which was almost a two year advance but still one year below her school status.

Leo made a nine-month grade gain in arithmetic according to his total scores. He showed a half-year's growth in addition, a year's growth in division, and a growth of a year and one-half in subtraction and multiplication. In his total reading score he regressed one month. A gain of one month in

paragraph comprehension was overcome by a three-month's regression in word comprehension.

Mary made an arithmetic grade gain of two months in her total arithmetic score. She was still ungradable in addition and at the same level (H-2) in subtraction. In her total reading score a gain of seven months was noted. This placed her three months in advance of her school grade. A one-year grade growth was made in word comprehension.

Paul made a five month grade gain in arithmetic. He gained one year in addition, remained in grade H-2 in subtraction, and reached H-2 in multiplication in which he was ungradable at the beginning of the term. He gained very slightly in reading but was still ungradable in reading in the final tests.

5. Conclusions Concerning the Efficiency of the Remedial Measures

It was not the purpose of this study to measure the relative efficiency of the remedial procedures used in this teaching. It was the purpose to determine to what extent the difficulties could be modified or removed. The teacher realized that some of the procedures were not approved by all educational authorities for general classroom use. With this highly individual work they were apparently fruitful. The first two of the procedures listed below were considered by the teacher to be the most vital.

1. Assignment cards made daily for each child and the selection of materials assigned influenced by the needs of the preceding day and by the criteria for good drill discussed in another portion of this paper.

2. Daily informal meetings with the teacher in which the successes and failures of the previous day were discussed. During this meeting an effort was made to make the child feel contented, and to inspire him to use his best honest effort.

3. Oral drill.

4. Written drill.

5. Flash card work.

6. Student's hearing each other when their work was finished.

7. Self-competition.

8. Friendly competition in pairs.

6. Conjectures Concerning the Contribution of the Schools to Arithmetic Handicaps

It might, perhaps, be better in this connection to speak of conjectures rather than conclusions. The writer feels that objective evidence has been presented in support of any statement made concerning either the behavior or the achievement of the six children involved in this study. No claim, however, is made that factual evidence exists concerning the part played by the schools from which the children came in the develop-

ment of their several handicaps. But close contact with the children over a period of six weeks resulted in some knowledge of their past school experience, on the basis of which the writer suggests as hypotheses, worthy, at least of consideration, the following:

The chief causes of children's failures and handicaps in arithmetic, from the standpoint of the school, are:

1. Failure of teachers to recognize each step of every fundamental process.
2. Failure of teachers to realize that the number combinations are not of equal difficulty.
3. Failure of teachers to consider the criteria for good drill.
4. Failure of teachers to balance their work so that overlearning of some facts and underlearning of others could be avoided.
5. Failure of teachers to demand automatism rather than "reasoning" of the combinations.
6. Failure of many textbooks to present appropriate and scientifically distributed drill.
7. Failure of teachers to recognize certain limitations in the power of transfer.

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The thesis "A Study of the Improvement in Computation of Six Children During a Six-Weeks' Term of Individual Remedial Work," written by Ethel Keewan Harrington, has been accepted by the Graduate School of Loyola University with reference to form, and by the readers whose names appear below with reference to content. It is, therefore, accepted as a partial fulfilment of the requirements of the degree.

John W. Scanlan

May, 1933

Austin G. Schmidt, S. J.

May, 1933